BIOGRAPHICAL
MEMOIRS

Fredrik Carl Mülertz Störmer, 1874-1957

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Biogr. Mems Fell. R. Soc. 1958 **4**, 257-279, published 1 November 1958

Carl Størmer

FREDRIK CARL MÜLERTZ STÖRMER

1874-1957

1. Introduction

This biography records the life of Carl Störmer, a son of Norway who added many important scientific achievements to his country's laurels. Born on 3 September 1874 at Skien, Norway, he died at Blindern on 13 August 1957, nearly 83 years old. I have no record of his ancestry or family background, but material concerning his childhood is given in an interview published in 1925 in a journal (*Varden*) not available to me.

In March 1956 he sent me a brief list of 'biographical notices' about himself—perhaps realizing that I might be invited to write such a notice as this. It contained several interesting particulars herein incorporated, which otherwise would not have come to my notice. In this typed list the o in his surname is written ö, not ø.

He was the only child of Georg Ludvig Störmer (an apothecary or dispensing chemist, first in Skien, later in Oslo), and Henriette Störmer, née Mülertz. From his childhood he showed a deep interest in the natural sciences, astronomy, physics, chemistry, meteorology, geology and in particular botany. At the age of about sixteen his interest turned exclusively to pure mathematics.

He was a student at the national university at Christiania (now Oslo) from 1892 to 1897; in 1898 he became a candidatus realium (roughly on the academic level of a Ph.D., but not carrying the title of Doctor); he gained a university stipendium (a five year research fellowship) in 1899. He continued his mathematical studies at the Sorbonne in Paris from 1898-1900, and at Göttingen in 1902. At the Sorbonne he studied under Picard, Poincaré, Painlevé, Jordan, Darboux and Goursat. He gained a good knowledge of the French and German languages.

In 1900 he married Ada Clauson, who survives him. They had five children. Precocious as a mathematician, he published original papers on the subject from his earliest student years—one in 1892 (aged 18) on the summation of trigonometric series, three more in 1895, and a further three in 1896. In each of these last two years, also, he published short botanical notes—tokens of an interest in living Nature that persisted throughout his life. I recall that when he visited me in London in 1930 to give a short series of lectures at the Imperial College, he brought with him a botanist's metal collecting case, hoping to find flowers in England that were new to him; the month, unfortunately, was unpromising—February—and the only

flowers readily available were those of groundsel, of which, he averred, Norway had an ample supply. In 1944 Knut Thalberg produced a list of Störmer's papers up to 1943, partly based on Störmer's own records; this list includes further botanical papers in 1925 and 1943.

Störmer continued to produce papers on pure mathematics: one in 1897, another in 1898, two in 1899, two in 1900, one in 1901, and in 1902 as many as five. Their subjects were series, theory of numbers and theory of functions. In 1902 he also helped to edit (with Holst and Sylow) two memorial volumes to celebrate the centenary of the birth of Norway's short-lived mathematical genius Niels Henrik Abel—a Festschrift in Norwegian (374 pages 4°) with a translation in French (460 pages 4°); he also prepared a study, published in Norwegian (62 pages) and in French, of documents left by Abel; and with Guldberg he edited a posthumous paper by Sophus Lie on integral invariants and differential equations. In these editions, and in general throughout his life, he gave his name as Carl Störmer, omitting his first and third names.

In 1903, at the age of 29, he was appointed professor of pure mathematics at Oslo—a position he occupied for 43 years, retiring in 1946 at the age of 72. By enquiry of Professor S. Rosseland, I learn that there was one other candidate, Guldberg. The expert committee for the appointment consisted of J. Petersen and H. G. Zeuthen (Copenhagen), F. Klein (Göttingen), E. Picard (Paris) and L. Sylow (Christiania). Zeuthen, Picard and Sylow preferred Störmer, the other two were neutral. 'After some haggling in the Faculty', the candidates were placed in the order (1) Störmer, (2) Guldberg; and the government acted on this basis. I have found reason to believe that present-day pure mathematicians would not dissent from this view of the relative merits of the two candidates, as pure mathematicians. Certainly Norway must be congratulated on providing Störmer, by this appointment, with a good economic foundation for his life-long researches, that brought fame to him and his country. He gave a grand survey of his major works in his book *The polar aurora* published in 1955.

These, however, proved to be very different from what was to be expected from Störmer's work up to that time. Pure mathematics was not left out of his thoughts. In 1903 he published three more mathematical papers, and a note on a letter from Abel to Külp. Further brief notes on pure mathematics appeared in 1908, 1933, 1937, 1938, 1940, 1942, 1943, and 1944. He lectured on many topics in pure mathematics, and produced stencilled notes on some of his courses, in 1924, 1932, 1933, and 1942. He occasionally reviewed mathematical books, and he wrote many biographical notes on mathematicians, such as Abel, Lie, Darboux, Holst, Sylow, Thue, Jordan, Mittag-Leffler, and Ramanujan.

Professor L. J. Mordell informs me that Störmer was expert in number theory and allied subjects. The mathematical problems he dealt with are of considerable interest and importance. Their solution was not easy, and his methods were elegant. In 1895 he showed that there are only four non-

trivial solutions of the equation $m\tan^{-1}(1/x)+n\tan^{-1}(1/y) = k\pi/4$, where m, n, k, x, y are integers. He returned to this subject in 1943 after his results had been extended by others to allied problems. This subject led him in 1897 to several problems connected with the Pellian equation $x^2-Dy^2 = \pm 1$; he completely solved the problem of determining when every prime p dividing y also divides D. 'His result is very pretty, and there are many applications of it.' Two papers (1900, 1902) relate to Diophantine approximation, e.g. the question of approximating to an irrational number ξ by means of a rational fraction x/y. Thus if t is a given number, what can be said about a function $f(t)$ such that there is an integral solution in x, y of $|x/y-\xi|<f(t)$, with $0<y<t$, or again when there is no solution for all t? Störmer considered the second problem in some entirely new cases, e.g. when $\xi = \log\alpha/\log\beta$, where α, β are algebraic numbers, and again when ξ is the quotient of two elliptic integrals. He showed that we can take for $f(t)$, $e^{-\omega t}$ and $e^{-\omega t^2}$ respectively, where ω depends only on ξ. Results of this type have attracted much attention in recent years, and Störmer's results are well known.

2. Trajectories of electric charges in the geomagnetic field

In the year of Störmer's appointment to the chair at Oslo, his colleague Professor Kr. Birkeland showed him a series of beautiful experiments on the movements of cathode rays in magnetic fields, particularly in the field of a magnetized sphere or magnetic dipole. The event permanently diverted Störmer's research interest from pure mathematics to auroral problems. Birkeland in 1896 had proposed that auroras are caused by electrons shot from the sun, and deflected to high latitudes by the earth's magnetic field. Poincaré in 1896 had integrated the equations of motion of a charged particle in the field of a magnetic *pole*. Their work impelled Störmer to study the more complicated and realistic problem of the motion of a charged particle in the field of a magnetic point *dipole*. His first contribution on the subject formed his only paper published in 1904.

In 1905 he became a co-editor of *Acta Mathematica*; his only publication that year was a 31-page description of the scientific papers left by Sophus Lie. But the publication in 1906 of three short papers in the Paris *Comptes Rendus* on electron trajectories in the geomagnetic field, and on Villard's experiments on such orbits, gave a foretaste of the results of his active work on this difficult problem.

The first extensive exposition of these researches was given in 1907, in a series of papers (in French) in the *Archives des sciences physiques et naturelles de Genève*. He found first integrals of the equations of motion of a charged particle moving solely under the influence of the field of a stationary magnetic dipole. The speed is clearly constant. From the first integrals he was able to draw valuable and illuminating inferences concerning the nature of the trajectories. The complete solutions required numerical integration. Over many years he made or organized such calculations, finding a great variety

of orbits. His papers on this subject constitute a major part of his life's work; they number at least 48. Many are short, and in several cases he published brief accounts of the same work in different languages and journals. But many of the papers are very substantial in length and content.

They fall into a number of series. One (A) began with the Geneva papers of 1907 (numbering 137 pages in all); it continued in the same journal in 1911 (101 pages) and 1912 (55 pages); he also published a number of shorter notes there at different times. This series was entitled 'Sur les trajectoires des corpuscules électrisés dans l'espace sous l'action du magnétisme terrestre avec application aux aurores boréales'.

Another series of four papers (B) began in 1907, and was continued in 1913 and 1916 (two papers); the title was 'Sur un probleme relatif au mouvement des corpuscules électriques dans l'espace cosmique'. It was published in the *Skrifter* of the Oslo Academy, which in 1912 and 1916 also contained two papers (series B^1) entitled 'Quelques théorèmes généraux sur le mouvement d'un corpuscule électrique dans un champ magnétique'.

Overlapping with these series (B, B^1), a new series (C), which also appeared in the *Skrifter*, began in 1913, under the title 'Résultats des calculs numériques des trajectoires des corpuscules électriques dans le champ d'un aimant élémentaire'. This series consisted of eight extensive memoirs. Long intervals intervened between the first three (1913) and the next two (1936), and again between the appearance of these and the final three (1947, 1949 and 1950).

The first long gap in the succession of the trajectory papers extended from 1916 to 1930. In the meantime, E. Brüche had repeated and further developed Birkeland's experiments on beams of cathode rays projected into a magnetic dipole field; his admirable photographs of their form strikingly confirmed some of Störmer's theoretically calculated paths. This work has more recently been continued by W. H. Bennett, who gave to his excellent equipment for the purpose the name Störmerton.

Moreover Störmer became aware of the existence of certain short period pulsations of the geomagnetic field, which Eschenhagen was among the first to recognize as worldwide in extent; more recently they have been studied by Grenet, Kato, Kalashnikov, Troitskaya and others; the periods range from a few seconds to some minutes. He thought that these pulsations might be explained by the magnetic field of electrons moving in periodic orbits such as those whose existence had been revealed by some of his calculations. These developments inspired the production in 1930 of a paper in the *Zeitschrift für Astrophysik* entitled 'Periodische Elektronenbahnen im Felde eines Elementarmagneten und ihre Anwendung auf Brüches Modellversuche und auf Eschenhagens Elementarwellen des Erdmagnetismus'. This was followed in 1931 by a paper in *Terrestrial Magnetism*, entitled 'On pulsations of terrestrial magnetism and their possible explanation by periodic orbits of corpuscular rays'. The years 1931 and 1932 saw the publication in the *Zeitschrift für Astrophysik* of a series (D) of three papers on 'Ein Fundamental-

problem der Bewegung einer elektrisch geladenen Korpuskel im kosmischen Raume'.

Another development that closely concerned Störmer's work on the trajectories was the growth of interest in cosmic rays among many physicists. In 1931 he drew attention to the application of his theoretical papers to cosmic rays. In 1933 Rosseland, on his return from Boston, Mass., U.S.A., brought to Störmer's notice the recent work on cosmic ray paths by Lemaitre and Vallarta, performed on the differential analyzer at the Massachusetts Institute of Technology. This stimulated Störmer to publish in 1934 in the *Physical Review* a critical commentary on the paper by Lemaitre and Vallarta; rather too severely he noted some errors therein, but he did not realize the importance of their application of Liouville's theorem to these problems. In 1933 he began a new series (E) of six papers entitled 'On the trajectories of electric particles in the field of a magnetic dipole with applications to the theory of cosmic radiation'; the series was continued in 1934 (two papers) and in the years 1935, 1936, 1937. The first two appeared in the Oslo *Avhandlinger*, the remainder in *Astrophysica Norvegica*.

3. Outline of Störmer's main 'trajectory' results

The several long series of papers briefly recounted in § 2 almost all dealt with the trajectories of a particle of mass m, charge $-e$(e.m.u.) and velocity $\mathbf{v}$ in the field $\mathbf{H}$ of a point magnetic dipole of moment $\mathbf{M}$. The vector equation of motion has the simple form

$$m\dot{\mathbf{v}} = -e\mathbf{v} \times \mathbf{H}, \tag{1}$$

where x is the symbol for the vector product operation on two vectors. As the force is perpendicular to the velocity, the speed v is constant. This is one 'first integral' of the equation.

Störmer simplified the equation by adopting C_{st} as the unit of length, where

$$C_{st} = (Me/mv)^{\frac{1}{2}}. \tag{2}$$

He took the origin at the dipole, and the z axis along the direction of $-\mathbf{M}$. He used both spherical and cylindrical polar co-ordinates, r, θ, ϕ and R, z, ϕ, so that

$$r^2 = R^2 + z^2. \tag{3}$$

Let an accent over a symbol denote differentiation with respect to the path length s along the trajectory, and let a dot denote partial differentiation with respect to R.

The equations of motion can be written

$$R'' = R(\phi')^2 + (r^2 - 3z^2)R\phi'/r^5, \tag{4}$$
$$z'' = 3zR^2\phi'/r^5, \tag{5}$$

and a third equation which is integrable, with the solution

$$\phi' = 2\gamma/R^2 + 1/r^3, \tag{6}$$

where γ denotes a constant of integration, that can have any positive or negative (or zero) value; (6) is another first integral of (1).

If θ is the angle between the tangent to the trajectory, and the meridian plane through the particle, so that $\sin\theta = R\phi'$, it follows from (6) that

$$\sin\theta = 2\gamma/R + R/r^3.$$

This equation gives the value of γ for the trajectory through any chosen point R, z for a given value of θ. But since

$$-1 \leq \sin\theta \leq 1,$$

the trajectories cannot go through the points in space where $2\gamma/R + R/r^3$ is greater than 1 or less than -1. This condition defines boundaries between the regions that are *forbidden* to the trajectories and the regions in which trajectories are *allowed*. The boundaries are in all cases surfaces of revolution about the dipole axis. The meridian sections of these surfaces for $\gamma < -1$ have three branches; two are ovals which at the origin, where they meet, are pointed, the third is a curve extending to infinity at both ends. These curves divide space into two separated allowed regions.

Only if $-1 < \gamma < 0$ does the allowed space stretch without interruption from infinity to the origin (which is a singular point of the differential equations, of a very complicated kind). For *small positive* values of γ the meridian sections of the allowed regions have 'horns' projecting towards the dipole, above and below the plane $z = 0$.

From these equations z can be obtained in the form

$$z = f\ (R, C_1, C_2), \tag{7}$$

where C_1 and C_2 are two constants of integration, which arise in the solution of an equation of the form

$$\ddot{z} = q_0 + q_1\dot{z} + q_2(\dot{z})^2 + q_3(\dot{z})^3, \tag{8}$$

where the q's are functions of R, z and γ. Using (7), it is possible by a quadrature to obtain s as a function of R; finally ϕ is found by a further quadrature. Thus the paths are completely determined by this procedure.

The paths in the equatorial plane of the dipole, $z = 0$, can be treated more simply, and expressed in terms of elliptic functions. Störmer showed that their radius of curvature is equal to R^3. The paths include some that come from infinity, and others that are periodic. They include a circle, an equilateral hyperbola, and the z axis. These are the only *algebraic* curves in the whole family of trajectories.

Störmer made another transformation of the equations by introducing new variables, eliminating the constant γ. The new variables R_1, z_1, r_1 and τ, which replace R, z, r and s, are defined thus if γ is positive:

$$R_1 = 2\gamma R;\ z_1 = 2\gamma z;\ r_1 = 2\gamma r;\ \tau = 8\gamma^3 s. \tag{9}$$

If γ is negative, a minus sign is inserted on the right of each equation in (9).

When γ is positive, let

$$U = -\left(\frac{1}{R_1}+\frac{R_1}{r_1^3}\right)^2 \tag{10}$$

The equations of motion can be written thus:

$$\frac{d^2R_1}{d\tau^2} = \frac{1}{2}\frac{\partial U}{\partial R_1},\ \frac{d^2z_1}{d\tau^2} = \frac{1}{2}\frac{\partial U}{\partial z_1},\ \left(\frac{dR_1}{d\tau}\right)^2+\left(\frac{dz_1}{d\tau}\right)^2 = U+W_0^2, \tag{11}$$

where $W_0 = 1/4\gamma^2$.

The same set of equations applies when γ is negative, except that in this case

$$U = -\left(-\frac{1}{R_1}+\frac{R_1}{r_1^3}\right)^2. \tag{12}$$

When $\gamma = 0$, one obtains the same set of equations involving R, r, z, s instead of R_1, etc., if

$$U = -R^2/r^6,\ W_0 = 1. \tag{13}$$

Thus instead of studying the equations (4 to 6) for an infinite set of values of γ, only the set (11), in its three forms, needs consideration; this is a most important reduction, obtained by Störmer in 1913.

He made yet another transformation, which simplifies the numerical calculation of the trajectories. It proved useful in the machine calculations by Lemaitre and Vallarta in their cosmic ray studies. Let

$$R+iz = e^{u+iw},\ ds = e^{2u}\ d\sigma, \tag{14}$$

and let the dot over a symbol now denote $d/d\sigma$. Then the equations of motion may be transformed to

$$\ddot{u} = e^{2u}+2\gamma e^{-u}+e^{-2u}\cos^2 w, \tag{15a}$$

$$\ddot{w} = e^{-2u}\sin w\cos w - 4\gamma^2 \sin w/\cos^3 w, \tag{15b}$$

$$\dot{u}^2+\dot{w}^2 = e^{2u} - 4\gamma e^{-u} - 4\gamma^2/\cos^2 w - e^{-2u}\cos^2 w, \tag{15c}$$

$$\dot{\phi} = 2\gamma/\cos^2 w+e^{-u}. \tag{15d}$$

Here $R = 0$, $z = 0$ correspond to $u = -\infty$, $w = \pm(\frac{1}{2})\pi$; that is, the

singular point (the origin) is removed to infinity, as is desirable in the numerical work.

Defining P by

$$P = e^{2u} - 4\gamma e^{-u} - 4\gamma^2/\cos^2 w - e^{-2u}\cos^2 w, \tag{16}$$

the system of equations can be expressed thus:

$$\ddot{u} = (\tfrac{1}{2})\,\partial P/\partial u,\ \ddot{w} = (\tfrac{1}{2})\,\partial P/\partial w,\ \dot{u}^2+\dot{w}^2 = P. \tag{17}$$

Similar equations may be obtained in variables u, w, σ similarly defined in terms of R_1, z_1, τ for the three cases γ positive, negative or zero, relative to slightly different forms of P.

The equation (15*d*) may be transformed by introducing a new variable ω given by

$$\tan\omega = dw/du,$$

that is, ω is the angle between the tangent to the integral curve and the axis of abscissa, if u is chosen as abscissa and w as ordinate. Then (15, *a*, *b*, *c*) become

$$\dot{u} = K\cos\omega,\ \dot{w} = K\sin\omega,\ \dot{\omega} = (\partial K/\partial w)\cos\omega - (\partial K/\partial u)\sin\omega,$$

where $K = \sqrt{P}$.

If u, w and ω are interpreted as Cartesian co-ordinates in space, these three equations give the direction of the tangent to the integral curve at each point. This yields a method of discussing the curves and the actual trajectories.

These transformations are set out in more detail in Chapter I of Part II of Störmer's book *The polar aurora*. In Chapter II he gives some main inferences drawn from the direct study of the differential equations.

The immense calculations by Störmer and his assistants, to explore the great variety of possible orbits, were extended by Lemaitre and Vallarta, with special reference to cosmic ray problems. Alfvén, using a perturbation method, obtained curves that in many cases approximate closely, in certain regions, to the true trajectories, and can be found with far less trouble of computation.

Störmer illustrated many of his results by wire models and by drawings. Close correspondence was found between his trajectories and the paths of narrow bundles of cathode rays in a dipole magnetic field, in the experiments by Birkeland, Brüche, Malmfors, Bennett and Brunberg. Störmer suggested that such experiments might indicate the trajectory shape, in some cases, more quickly than by numerical computation.

In all this work a fundamental parameter is the rigidity of the particle in relation to the magnetic field. The stiffness is defined as $H\rho$, where H denotes the magnetic intensity, and ρ the radius of curvature of the trajectory at the point:

$$H\rho = mv/(-e)$$

This is related to the Störmer length unit as follows:

$$C_{st} = (M/H\rho)^{\frac{1}{2}}\text{cm}.$$

4. Störmer's applications of his calculations to the actual aurora

Störmer considered that his mathematical and computational work on the particle trajectories gave promise of explaining many features of actual auroras. He considered alternative possibilities regarding the particles—they might be electrons or protons. For a range of the rigidity $H\rho$ from 100 to a million, the particle energy in electron volts ranges from 880 to 3×10^8 for electrons, and from 0·48 to 48 million for protons. The Störmer distance correspondingly decreases from 9 million to 90 000 km, the same for either kind of particle.

Störmer's theory of forbidden regions indicated very simply that a particle entering the earth's atmosphere, and coming from a distance greater than C_{st}, cannot approach the magnetic pole nearer than a limiting north polar angle θ. This he associated with the poleward limit of the auroral zone. 'Here we encounter the first disagreement between theory and observation. For cathode rays the auroral region is too near the magnetic axis, and even for protons the region of maximum frequency is further south than the theoretical one. For aurorae during violent magnetic storms, the difference is even greater, as the aurora in these cases may be observed overhead even in central Europe' (*Polar aurora*, p. 296).

In his second Geneva paper (published in two parts, in 1911 and 1912), Störmer 'tried to remove the difficulty of explaining the real radius of the aurora zone by admitting an action on the moving corpuscle of a stream of other corpuscles round the earth . . .' (*Polar aurora*, p. 340). Negative particles shot towards the earth from the sun, in the plane of the dipole equator, tend 'to bend round the magnetic globe on the evening and night side . . . and even to encircle it like a ring. On the morning and noon side, however, they are thrown away from the geomagnetic equatorial plane' (*ibid*, p. 341). Störmer investigated the magnetic field of such electron streams, treating them as a complete ring round the earth (thus making a mathematical approximation legitimate in such an exploratory study). The electric current flow is westward. He found that the magnetic field of such a ring current could draw the aurora zone nearer to the tropics; the same effect was experimentally demonstrated by Brüche in 1930.

This work is the origin of the 'Störmer ring current', still often referred to. Its dimensions, as given in his Geneva paper, located it beyond the moon's orbit. The magnetic field needed to move the aurora zone down to the latitudes sometimes reached, would profoundly modify the geomagnetic field over an extensive volume of space around the ring current, though causing no more than the observed magnetic storm changes near the earth's equator. The field near the ring would reverse the dipole field there. The ring was a mathematical idealization of electron streams mainly sweeping partly round the earth.

In *Polar aurora* Störmer did not reproduce his Geneva specifications of the ring current, and attached no decisive importance to his conclusions, though regarding them as pointing in the right direction (p. 345).

More recent speculations on the theory of magnetic storms and auroras, by Ferraro and myself and others, still contemplate the possibility of a ring current, but of a size and kind quite different from those above mentioned; the current is tentatively located within a few earth radii from the earth's centre, and there it only fractionally alters the dipole field. The existence of a ring current, inferred from purely geomagnetic evidence by Ad. Schmidt in 1916, still remains a speculation; no clear indication of its mode of formation has yet been given. Satellite observations may ere long confirm or refute the current conceptions on this subject.

Störmer tried to explain the limitation of the auroral zone on the side towards the pole by reason of (a) the origin of the particles in the sun, whose declination has a limited range, between $\pm 23\frac{1}{2}°$ relative to the geographical axis, or (what is here more relevant) between $\pm 35°$ relative to the geomagnetic axis; and (b) of the nocturnal appearance of the aurora, because trajectories that meet the earth on the side away from the sun correspond to γ less than about $-0 \cdot 5$. Störmer recognized (p. 298, *loc. cit.*) that also this explanation met with difficulties. 'It may be possible that' auroras seen in high geomagnetic latitudes 'can come from sources other than the sun', or that the simplifying assumptions on which his theory was based are too restrictive.

He showed that the particles issuing from the sun in a narrow cone can impinge on the night side of the earth within a narrow belt of geomagnetic latitude: he proposed this as the explanation of the frequent appearance of thin auroral arcs lying nearly in the direction magnetic east to west; he extended this explanation, with detailed numerical examples, to the case of folded auroral arcs.

In his 1912 Geneva paper (which contains a correction to his 1907 Geneva publications) he considered how the trajectories would be affected by taking into account the non-dipole terms in the geomagnetic potential.

5. Criticisms of Störmer's theoretical work on the aurora

One short chapter in Störmer's book *The polar aurora* is entitled 'What the mathematical theory can or cannot explain'. The meaning of *explain* as here used is perhaps ambiguous, but the chapter is modestly frank about what the theory cannot explain. It indicates clearly the simplifying hypotheses adopted at the outset of the theory. The most serious of these is the neglect of the action of the other particles on the single particle considered. Schuster criticized Birkeland's theory of aurora and magnetic storms on the ground that a stream of particles all having charges of the same sign would be dispersed by their mutual repulsion. Lindemann (later Lord Cherwell) proposed a neutral ionized stream as the cause of auroras and magnetic storms. This view is now generally adopted, and it has been shown by Ferraro and myself (1940), in an idealized problem, how low must be the density of a neutral ionized stream if it is valid to neglect, as in Störmer's

theory, the interactions of the particles; a stream of such low density could not produce the observed magnetic effects upon the earth.

To the writer it has long seemed that Störmer's theoretical work did not attain its main objective, and that auroral theory must be developed along radically different lines. Störmer himself seemed unmoved by such criticisms. His book *The polar aurora* is devoted almost wholly to his own work, and is a valuable unitary presentation thereof. He has, however, included a brief summary (p. 339) of an attempt by Vegard to meet Schuster's criticism, by supposing that the stream of solar particles, though electrically neutral, carries an electric current. An alternative attempt to meet Schuster's criticism has been given in recent years by Hulburt and Bennett, and criticized by Ferraro and by Vegard. In a late chapter of *The polar aurora* Störmer gave brief summaries of other theories of the aurora, with a few critical remarks.

The book has a good author's index and table of contents, but its subject index might, with advantage to its readers, have been much fuller.

6. Störmer's theory and cosmic rays

If Störmer's theoretical work did not reach his objectives, and omitted vital factors in auroral causation, it is the more fortunate that it turned out to have an important field of usefulness, originally totally unexpected, in connexion with cosmic rays. As he truly said, 'The objections raised against the validity of the application of this theory to the aurora, are not as serious when the theory is applied to cosmic rays. In fact, since the energy of the cosmic corpuscles is so great and their concentration in space so small, the action of streams of such corpuscles on each other may be neglected' (*The polar aurora*, p. 375). As already indicated, Störmer made numerous valuable contributions, both critical and constructive, to this subject.

7. Störmer's photographic auroral studies

Störmer's theoretical work on the aurora stimulated him to make observational studies, from 1909 onwards. This quite different part of his researches has earned for him an undying fame in the history of auroral science, quite independent of the theoretical work so far described. In *The polar aurora* he gave first place to the observed data about auroras, leaving the account of his theoretical work to Part II. He was an enthusiastic and indefatigable observer and organizer of observations. His last paper, that appeared in the year of his death, was an account of Norwegian auroral observations in the preceding year, 1956.

In his preface he remarked that in 1909 he 'found it necessary to obtain more facts about the aurora in order to compare theory and observation. A photographic method designed to determine, among other things, the height and position of aurora was therefore developed and successfully applied. The chief results obtained from the analysis of a vast number of parallactic photographs are discussed in this book.'

Though the aurora was first photographed in 1892, Störmer in 1909 began systematic experiments to assure easy and successful auroral photographs, as a preliminary to an auroral expedition he made to Bossekop in northern Norway in 1910. There he set up two stations for parallactic photography, to determine the height and location of the auroras that were so much more often visible at Bossekop than at Oslo. The base line between the two stations, 4¼ km, proved to be too small: the experience gained in this expedition was used in 1913 in a second expedition which he organized, also to Bossekop, this time using a base line of 27½ km. About 2500 determinations of height and position of selected points of the aurora were obtained. In all he obtained more than 40 000 auroral photographs.

From 1911 up to his last years Störmer continued this parallactic photography in southern Norway. He amassed more than 9000 sets of pictures, from which he derived the heights and locations of more than 18 000 auroral points. His work involved telephonic links with his co-operating observers, at the other ends of his base lines; and the Norwegian telephone authorities gave him good facilities of this kind. He used the help of many assistants during this long-continued work, both for the actual photography, and for the work of measurement and calculation from the plates.

Apart from the nocturnal effort involved in getting the photographs, their reduction was a herculean labour, in which Störmer showed the same perseverance and tenacity as he manifested in his theoretical work. In measuring his early Bossekop plates he developed a method of projecting the negatives on a wall: the magnified image facilitated identification and measurement of the same auroral points on two or more plates. At first the reductions from the measurements were made by numerical calculation, but gradually graphical methods were adopted. In the evolution of appropriate methods, Störmer's own ideas were supplemented by those of Vegard and Krogness, Harang and Tønsberg, Herlofson and Egeberg, most of whom acted for a time as his assistants. These methods are rather fully described in chapter 5 of *The polar aurora*.

The results of this work were described in a long series of extensive memoirs, mainly in the *Geofysiske Publikasjoner* of the Norwegian Academy. Störmer was the first to establish beyond doubt the height of the aurora, as to which, despite some notable earlier attempts, there was great uncertainty when he began his work. He gave valuable statistical discussions, and diagrams that summed up the results of his observations; these (and similar researches by others) are described in chapters 6 to 8 of *The polar aurora*. His work stimulated others in different parts of the earth to observe and calculate auroral heights and positions—notably during the second International Polar Year. He actively encouraged and helpfully advised others who took up such work; for many decades he was the acknowledged leader among auroral observers. He long acted as chairman of the auroral committee of the International Association of Terrestrial Magnetism and Electricity (now the International Association of Geomagnetism and Aeronomy) of the Inter-

national Union of Geodesy and Geophysics. In this capacity he prepared (1930) a valuable *Photographic atlas of auroral forms*, giving a classification of forms, indicated by numerous photographs by himself and his assistants; it also described methods of photographic and visual observation of auroras. Later, in conjunction with la Cour, he prepared a *Supplement* to this *Atlas*. He was President of the Auroral Committee for the Second International Polar Year 1932/33.

8. Synoptic auroral studies

The parallactic auroral photographs gave not only the heights of individual auroral features, but also their location in geographical plan. Störmer published many synoptic charts showing the plan location of many auroral arcs and other forms shown on his photographs. He also published combined synoptic charts, showing the plan location of whole series of arcs observed by him over several years. He showed that the arcs are extended neither along the geographic nor the geomagnetic circles of latitude, but that they lie nearer the latter than the former.

9. Special auroral forms and sunlit auroras

Störmer's studies of the heights of auroral points led him to provide valuable height-frequency diagrams, showing the height-range and height-distribution of auroras in general, and of many special types of aurora. His work forms the main basis of our knowledge in this field.

In the course of these photographic studies, in which he personally took a leading part, involving very many nights of long-continued work, he was quick to observe and distinguish auroras that departed from the commoner types. As a result he published many descriptions of special types of aurora, notable for their unusual form, movement, pulsation, or height. Among these fascinating researches, his discovery of the remarkable properties of sunlit auroras was the most prominent. It may be added that already in 1923 Vegard on theoretical grounds had inferred, and remarked to Störmer, that the upper limits of height of auroras should increase at lower latitudes (and he found observational evidence supporting this inference), and be greater at dawn and sunset twilight than at night.

In the great aurora of 22/23 March 1920 (of which he gave a most interesting description *The polar aurora*, pp. 8, 9) he found that in the earliest and latest periods of the night the auroral rays extended to very unusual heights, from bases at 250 to 350 km height, up to more than 800 km, much more than the auroral heights observed during the middle of the night. The significance of this difference was not realized until 6½ years later, when he studied another remarkable grey-violet aurora (8 September 1926). It occurred to him that the remarkably high rays then also observed, after sunset, might be situated in the still sunlit high atmosphere; and the measurements from his photographs confirmed that this was so. He went

back to his 1920 photographs and found there the same result. 'The astonishingly high rays after sunset and before sunrise were sunlit, and the lower rays were in shadow' (p. 127). He pursued his studies into these remarkable phenomena, which, he states, when once seen are easy to recognize, from 'their colour, their length and their occurrence after sunset or before sunrise' (p. 131). 'The colour is mostly feeble white-violet, but it may also be red or blue' (p. 131). Sometimes there is a clear difference in colour between the sunlit and shadowed parts of the same auroral rays. Sometimes also a ray will be broken, a part being missing over a height interval that straddles the earth's shadow line. These and other results, illustrated by a number of well-conceived diagrams, form the subject of chapter 9 of *The polar aurora.*

10. The colour and spectrum of the aurora

Störmer's descriptions of auroras give much information about auroral colours, of which he seems to have been a gifted observer. Though trained as a mathematician, his vital interest in observation led him to make spectroscopic studies of the aurora, but to a far less extent than did his colleague (and one-time assistant) Professor L. Vegard. *The polar aurora* includes a chapter (12) on the auroral spectrum, summarizing, though very briefly, the knowledge gained in this field, mostly by other workers, but also mentioning his own work, and that of some of his assistants.

It was Störmer's habit to carry with him a pocket spectroscope with which to examine the night sky, to seek for spectral evidence of the presence of aurora (even when the sky was cloudy), and to observe the main spectral features of coloured auroras.

He also kept by him the diagrams (prepared by J. Bartels) of the 3-hour *K* indices of geomagnetic disturbance, as a guide (based on the 27-day recurrence tendency) to the likelihood of auroral appearance on each evening.

11. Auroral sounds

Anyone who visits high latitudes is likely to meet 'old inhabitants' (and others) who assert that they have heard distinctive sounds during auroras. Usually they recall it as a rare happening, associated only with one or a few of the most outstanding auroras they have seen. Some scientists dismiss these stories as based on fancy or illusion, or as due to crackling of ice crystals frozen from the moisture of the observer's breath; they point to Störmer's own determination of the lower limit of height of the aurora—usually about 60 miles—and regard it as impossible that a phenomenon occurring so far above us should produce audible sounds.

Störmer himself did not enter into controversy on this subject. Apparently he, like the writer of this account, never himself heard the sounds. But he became convinced of their reality, from reports on two occasions of exceptional aurora: 15/16 October 1926 and 25/26 January 1938. The sounds

were heard by different observers (in 1938 by three persons) in whom he had confidence. He describes these cases on pp. 10 and 137-139 of *The polar aurora*. He added the comment: 'As the aurora on the two occasions was about 90 to 100 km overhead, the sound could not come directly from this, but must have been a secondary phenomenon: perhaps some sort of electrical discharge in the neighbourhood of the observers caused by the enormous amount of electricity accumulated in the region of the aurora. New observations of the electric potential gradient at the place of observation are needed to find if certain conditions were present, which do not accompany common aurorae of less intensity, even if the aurora is in the zenith' (p. 139).

Such speculations seem less improbable now that observations during the International Geophysical Year have revealed that ionizing X-rays associated with strong auroras can be detected down to levels of 20 miles or less. There is need for long continued audio and electrical recording in auroral regions, to gain fully objective records bearing on such reports.

12. Long delayed radio echoes

Besides being alert to recognize new or specially interesting auroral features, Störmer was quick to note new phenomena that seemed to bear on his auroral interests. One chapter (15) of *The polar aurora* is devoted to what soon became known as 'wireless echoes of long delay'. On this subject he wrote eight notes or papers, the first two in 1928, four more in 1929, and two in 1930.

The story begins with a chance meeting in December 1927 between him and his neighbour Hals, an engineer who occasionally reported to him apparent auroral influences on radio reception. Hals told of strange echoes he had heard of signals from the Dutch station Eindhoven; the echoes had a delay of about 3 seconds. Hals suggested that they came from the moon. Störmer at once thought of another possibility, that they came from the electrons beyond the boundary of his 'forbidden regions'; their surface would vastly exceed that of the moon, and they would be like a concave mirror, instead of convex like the moon.

Störmer and Hals then began to study and record the echoes in detail. They concluded that the sounds heard were indubitable echoes of the signals; and van der Pol confirmed their conclusion. Special signals were emitted to further the investigation, and echo delays were recorded ranging from 6 seconds up to as much as 15 seconds. The wave length of the signals was about 30 metres. 'The signals were so powerful that they hurt our ears, and the echoes were also very strong, even though they were far from being as powerful as the signals' (p. 175).

The echoes disappeared completely at times, for weeks or months. 'This was a very depressing fact, but a closer study of the mathematical conditions for the appearance of the reflexion from electron surfaces gave a satisfactory explanation' (p. 177). Störmer predicted that the conditions for hearing

echoes would recur in mid-February 1929, and his prediction was verified: for example, Appleton and Barrow in London observed echoes with delay times up to 25 seconds. In May 1929 many such echoes were heard at a small station in Indo-China.

Störmer's explanation of the echoes was not universally accepted; van der Pol, for example, regarded them as a phenomenon of the earth's ionosphere.

After 1930 the Eindhoven station could not send further signals. During the years 1947-9, Budden and Yates at Cambridge sought to detect long-delayed echoes from a station in Essex, England (wavelengths 22 and 14 metres). No echoes were heard.

Recently Ratcliffe has stated that such echoes are never likely to be heard again, because nowadays the air is too full of radio signals.

13. Other luminous phenomena of the atmosphere

Störmer's watch on the sky for aurora was occasionally rewarded by the observation of other interesting luminosities in the atmosphere. Among these were two kinds of cloud, occurring at unusual heights in the atmosphere.

One kind is called mother-of-pearl (or nacreous) clouds—a subject on which he wrote no less than seventeen papers from 1927 onwards until 1951. He applied his organization for determining auroral heights to the measurement of the heights of these clouds, and proved that they lie in the stratosphere, at heights from 21 to 30 km, well above the tropopause. They are thought to be formed of super-cooled water drops. They show beautiful iridescent colours, and in 1948 he presented colour photographs of them to the International Association of Meteorology, meeting at Oslo. He found that these clouds are associated with particular meteorological conditions (föhn), and that their occurrence in time was very irregular: Störmer saw them in 1890, but saw none between 1892 and 1926. He suggested that they may occur more often, but that in general lower clouds may hide them from view. The observation of these clouds is recommended to auroral workers as part of the programme of the International Geophysical Year.

Another type of cloud, occurring still higher in the atmosphere, bears the name *luminous night cloud*. These also Störmer observed, and again made good determinations of their height, which is about 80 to 85 km. Their nature is still uncertain. Störmer wrote eight papers on this subject between 1932 and 1942.

In 1911 he published photographs of the zodiacal light, and in 1929, 1939 and 1947 he wrote on meteor trails. His height-determining organization could not be applied to the study of such unexpected and fleeting luminosities as meteor trails, but he collected photographs that happened to be taken of certain trails. These he used to find their height and study the high-level winds indicated by the rapid distortion of the trails.

The polar aurora does not refer to the researches described in this section.

14. Störmer's interest in photography

In the preface to *The polar aurora* he remarked that 'it might be a source of interest to many to observe, in the development of my photographic work, what may result when a pure mathematician happens to be also an enthusiastic amateur photographer'. Like Chaucer's clerke of Oxenforde, 'gladly would he learn and gladly teach' in this favourite field of his endeavours. Besides writing on the photographic art as applied to auroral studies, he wrote also more generally on photography for amateurs, including photography of stars and of a solar eclipse.

In 1930 he told me how he developed into a photographer. When he was a young man at Oslo University he fell in love with a lady whom he did not know and with whom he was too bashful to become acquainted. Wishing at least to have a picture of her, he decided that this was possible only by taking a photograph of her himself, without her knowing. So he procured a small camera that could be concealed under his coat, the exposure being made through his buttonhole. He succeeded in this difficult task. The love affair came to nothing; later the lady left Oslo for America and married. Years later she returned to Oslo. By this time he was famous. Meeting her, he told her of the incident. Later he applied his skill in photography (unsuspected by the subject) to obtain pictures also of many of the celebrities of Oslo of those days. Long afterwards, when he was nearing the age of 70, these photographs formed the subject of an exhibition in Oslo. He published illustrated accounts of it (totalling over 60 pages) in 1942 and 1943—under the title (translated): 'Carl Johan (Street) snapshots of famous people of the last fifty years.'

15. Other scientific activities and suggestions

The form of the polar coronal plumes on the sun suggests that they may be associated with the lines of force of a magnetic field. With this in mind, Störmer in 1910 made calculations to find the trajectories in space of charged particles emitted normally from the sun, in a dipole magnetic field. In 1911 he wrote two papers on coronal structure, and on Arrhenius's theory of it. He spent some time as a Research Associate at Mount Wilson Observatory, California, in 1912. There Hale had recently crowned his discovery of sunspot magnetism by finding evidence of the sun's general magnetic field. This visit led Störmer to publish (in the *Astrophysical Journal* for 1912) a paper on solar vortices. He constructed wire models of some of his calculated trajectories of solar particles. They show much resemblance to the form of the solar corona at some epochs (p. 384 and Fig. 213, *The polar aurora*). In 1922 he attempted to estimate the pole strength of the sun's magnetic field, from the coronal structure.

In connexion with his calculations of corpuscular trajectories, he suggested that astronomers might well try to discover any luminous phenomena

resembling aurorae on the dark side of Mercury and Venus, especially during magnetic storms (*The polar aurora*, p. 361).

He wrote on a halo, in 1940. In 1923 he wrote in Norwegian a popular book that ranged from stars to atoms. It appeared in at least six languages; the French title is *De l'espace a l'atome*.

16. Some personal characteristics and anecdotes

I first met Störmer in 1921, when I was making a cycling visit to Norway. I found him to be a rather large and bulky slow-moving man. He gave me a friendly welcome to his office, to the Royal Academy of Norway (where he read a paper) and to his home, where his table was graced by many children. He was a thorough family man. At his death he left 5 children, 3 daughters-in-law, 2 sons-in-law, 15 grandchildren (3 of them married) and 5 great-grandchildren.

He liked the pleasures of the table, good food and wine; he liked the theatre and the opera, and took with him opera glasses the better to observe the action and the danseuses.

Once in my office in London he was visited by a known mathematician who wished to meet him. After she had gone he remarked to me that he had no idea that she would be so 'temptating'. From London he visited other universities and lectured there. He particularly enjoyed a dinner party after one of these lectures, and told me of one of his neighbours at table. Rather naughtily I asked if she too was temptating. 'No', he answered, 'not temptating, but cosy to talk to.'

He travelled much: to universities in other countries as a research student, to arctic Norway as an auroral photographer and investigator, to America as a research associate of the Carnegie Institution of Washington at Mount Wilson, and on at least three other occasions, to many foreign capitals and other cities to mathematical and geophysical congresses. He was an honoured and valued leader in international science in his chosen fields of research. His last long scientific journey was to an auroral symposium at London, Ontario, Canada; this was in 1951, shortly before he completed his 77th year. In later years he suffered serious illnesses.

As professor his rather exigent relationship with his assistants was far from that companionable ease, that level give-and-take, sometimes found in English university departments and more often in American. It is remarkable that none of his papers bears any name other than his own, though their text often refers to contributions by his assistants to their methods or ideas; some of those contributions were really substantial.

He made two visits to England, in 1930 and 1947, to give lectures in London, Oxford and elsewhere. He was a good lecturer, and his varied and fruitful researches provided ample material for his talks. In 1947, before his Halley lecture at Oxford, the honorary degree of D.Sc. was conferred upon him. He was photographed in his doctor's robes. Later—one small example

of his innocent delight in public recognition, and also of his meticulousness—he sent for a sample of the cloth of the robe, so that the photograph could be coloured with exactly the right shade.

The University of Copenhagen and the Sorbonne conferred a like honour upon him. In 1951 he was elected a foreign member of the Royal Society. He was a member of several academies of science in Scandinavia, and a Corresponding Member of the Paris Academy of Sciences, which already in 1922 had awarded him its Janssen Medal for his auroral researches. He was President of the International Congress of Mathematicians when it met at Oslo in 1936.

His marriage was long and happy. Mathematicians as husbands, with their reclusive habits of study, are not to the taste of every kind of wife. In his early married days, while he was studying in Paris, the only evening each week not dedicated to his studies was Tuesday. Later the pattern of his family life must have been considerably affected by his long nights' observations of the aurora, and by the watch he kept to seize favourable occasions. But as the preceding remarks will have indicated, he was very human, appreciative of family and neighbourly joys and the good things of life. He loved to amuse children and others with some tricks in which he was very adept. I particularly recall one in which he tied up a handkerchief to resemble a mouse. It seemed bursting with desire to escape from his restraining hand. Finally it succeeded in apparently running up his arm inside his sleeve, whence it was retrieved at the armpit from within his coat.

Dr Leiv Harang wrote, in a letter to me: 'Störmer always reminded me of the most lovable character in English literature: Mr Pickwick,—and when he lectured on his researches in auroras I always had to think of the scene in the *Pickwick Papers* where Mr Pickwick is presenting his report on his expedition to the ponds of London. I am thankful for having learned to know a man like Störmer; in our streamlined century a personality like his with all his peculiarities certainly never will develop.'

In 1950 Sir Edward Appleton, a General Editor for the Oxford International monographs on Radio, invited him to write a book on the aurora. He was then 75, and still active in auroral observation and study, though retired from his university chair. It was a tremendous task to undertake, but his acceptance and achievement of it was greatly to the benefit of his science. It finally appeared in 1955.

It is thus dedicated: 'To my wife ADA, who never ceased to encourage me to work hard till this book was safely finished.'

The book is limited in some directions, but it is and will be a classic in its field, worthy of the serious attention of all general students of the aurora. Its preface is written in the simple and modest terms worthy of a great scientist, such as he was.

In his reply to my letter of congratulation on its appearance, he remarked that everyone who engaged in the auroral programme of the International Geophysical Year ought to get a copy.

In the rooms of the Royal Astronomical Society in London there is a portrait of Gauss. Under the portrait there are the following words, put by Shakespeare in the mouth of the bastard Edmund in *King Lear*:

'... Thou Nature art my goddess,
To thy service I am bound.'

These words are as applicable to Störmer as to Gauss. Throughout his long life Störmer pursued his chosen course of observing Nature and applying his fine intellect to the unravelling of some of its mysteries. In the concluding paragraph of the preface to *The polar aurora* he says: 'My work has given me infinite pleasure and satisfaction, but I regard it as no more than a pioneer effort.'

The photograph of Störmer facing p. 257 was taken when he was in his forties.

S. Chapman

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