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Elected FRS 1978

BY ALLAN M. SINCLAIR

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Barry Johnson made major contributions to the theory of Banach algebras, stimulating
research on automatic continuity and cohomology in these algebras. His research on the con-
tinuous Hochschild cohomology of Banach and operator algebras led to major developments
in these areas, and to ‘amenability’ moving from a group theoretic idea to one that is widely
applicable in modern analysis.

EDUCATION AND ADMINISTRATION

Barry Johnson was born on 1 August 1937 in Woolwich, southeast London; the family moved
when he was young to Surrey, where he attended Epsom County Grammar School for Boys.
They emigrated to Hobart, Tasmania, late in 1951, where Barry went to Hobart High School
for the last two years of his school education. The parents and the two younger children of the
family returned to England in February 1954, leaving him in Hobart in the care of friends of
the family. Barry’s father was persuaded by the headmaster of Hobart High School to leave
him to attend the University of Tasmania, where he was a student from March 1953 to late
1957. The degree structure was based on the old Scottish model of a three-year ordinary
degree, mathematics with subsidiary physics and chemistry in Barry’s case, followed by an
additional year to give an honours degree in mathematics. Barry was an outstanding student
at school and university. There was no academic tradition in the family: his father was a skilled
toolmaker, and late in life a planning engineer; his mother was a secretary at Warner Bros.
While at the University of Tasmania, Barry completed his National Service during the vaca-
tions, which exempted him from National Service when he returned to England. He returned
to England at the end of his honours degree and taught at a grammar school in Tamworth
before going up to Cambridge. He was awarded a Rhondda Memorial Studentship, which
supported him while studying for a PhD at Gonville and Caius College, Cambridge University, supervised by John Williamson, starting in October 1958. Of this move to Cambridge, Barry wrote in 1978 that

This was probably the luckiest thing that happened to me because I was supervised by Professor J.H. Williamson who has a remarkable record as a supervisor. I met a number of research students who now have university teaching positions and also gained a lot of experience of life to transform me from a very dull bookish type to a more socially acceptable individual.

Barry also acknowledged his educational indebtedness to the Epsom and Hobart schools, and to the Department of Mathematics at the University of Tasmania. Several of Barry’s early research papers evolved from his thesis, ‘Centralisers in topological algebras’, which was examined by J.H. Williamson and J.R. Ringrose (FRS 1977).

After one-year lecturing posts at the universities of California, Berkeley (1961–62) and Yale (1962–63), and two years at Exeter he moved to the University of Newcastle upon Tyne in 1965. Except for periods on sabbatical leave in the USA he remained at Newcastle upon Tyne as lecturer, reader (1968) and professor (1969). Barry enjoyed lecturing at all levels from service teaching, even first-year agriculture students, to postgraduate students and would initially describe himself to total outsiders as a ‘teacher’. He supervised seven research students, of whom two are still currently active in mathematical research. At Newcastle upon Tyne he was at various times head of pure mathematics, head of the School of Mathematics and Dean of the Faculty of Science. He was active on many university committees and was the university representative on the governing body of the Newcastle Royal Grammar School for many years. On three occasions he was a visiting professor in the USA: Yale (1970–71), University of California, Los Angeles, and University of California, Santa Barbara (1990–91), and University of California, Berkeley (1999). He was elected a Fellow of The Royal Society in 1978.

Barry devoted considerable time and effort to administering mathematics and ensuring that its standards were maintained in the UK. He was on the Council of the London Mathematical Society, was its President between 1980 and 1982 and was editor of its Newsletter for four years. During the time of Barry’s presidency of the London Mathematical Society there was controversy over the Society’s position on the International Congress of Mathematicians in Warsaw while martial law was enforced in Poland, where some well-known mathematicians were prisoners of the regime. Barry believed that the London Mathematical Society should be non-political and that protest was an individual matter. He was a member of the Research Assessment Exercise Pure Mathematics panel in 1992 and chaired this panel in the 1996 Research Assessment Exercise. He served on the Science and Engineering Research Council mathematics committee and chaired its review panel of the Isaac Newton Institute in 1993. Barry did his full share of external examining, both of PhD students and final honours degree courses in the UK. In the late 1960s Barry Johnson made an outstanding contribution to running the North British Functional Analysis Seminar, which was founded by F.F. Bonsall (Edinburgh) (FRS 1970), J.R. Ringrose (Newcastle) and J.H. Williamson (York). This was the first UK inter-university seminar designed to support and encourage research in a single specialist area of mathematics across several universities. He ran the seminar for the first nine years and was instrumental in widening its scope from the initial three members so that currently there are 12 members, including some as far afield as Cambridge and Belfast.

Barry’s first marriage in 1961 to Jennifer Munday in Reno ended in separation in 1977, although he maintained a close relationship with his daughter and two sons. Shortly after this
he met Margaret Jones and lived happily with her for the rest of his life, becoming close to her three children by her previous marriage; they married in 1991 in Santa Barbara. Barry and Margaret are fondly remembered by many mathematicians for their parties and for the support that they provided to visitors to the department and at conferences.

Barry was a voracious reader, reading anything and everything: histories, biographies, novels and travel books. The other important thing in his life was walking regularly. He walked through many of his mathematics problems, and in any crisis would go off alone and walk. Barry also liked listening to music and doing his own repairs and alterations to their house and their cottage on the Scottish borders. Substantial DIY and woodwork were hobbies epitomized by a perfect scale model of their own house that he made as a dolls’ house for his daughter. His work and his private life were separate, with friends in one often unaware of his accomplishments in the other.

Barry is warmly remembered for his sharp lively humour, involving repartee and a keen eye for the ridiculous, which made him entertaining company for family and colleagues.

Barry developed cancer in February 2000 and struggled bravely with it through a major operation, radiotherapy and chemotherapy, with his life becoming physically more difficult. His research activity continued to the end of his life. A recently completed paper (70)* was found in his effects, and joint research was in progress (70, 72). He was able to attend some of the talks at an international conference to mark his scientific contributions and his retirement, which was held in Newcastle upon Tyne in June 2001. He clearly enjoyed seeing colleagues and old friends at the conference. He died in St Oswald’s Hospice, Newcastle upon Tyne, on Sunday, 5 May 2002.

MATHEMATICAL RESEARCH

Barry Johnson’s deepest and most influential work has been in two areas: the automatic continuity of isomorphisms, derivations and intertwining operators on Banach algebras, and the Hochschild cohomology of Banach algebras, $C^*$-algebras and von Neumann algebras. The discussion below is under these headings: automatic continuity, Hochschild cohomology and amenability of Banach algebras, perturbations in Banach algebras, derivations, and centralizers and other research. Of these his work on cohomology has had the most important effect on international research.

Automatic continuity (4, 6, 7–9, 14, 15, 18, 20, 21, 23, 38, 48, 51)

The idea behind automatic continuity is that certain natural algebraic equations satisfied by a linear operator between Banach spaces combine with the overall structure of the spaces to force the continuity of the linear operator. Presumably Barry became interested in these problems during his year in Yale (1962–63), where C.E. Rickart had worked on the problem of the uniqueness of norm; his first paper on automatic continuity was submitted from Yale. Following work of M. Eidelheit on $B(H)$ (Eidelheit 1940) and I. Gelfand (ForMemRS 1977) on commutative Banach algebras (Gelfand 1941), C.E. Rickart conjectured in (Rickart 1950) that there was a unique Banach algebra topology on a semisimple Banach algebra. I. Kaplansky had obtained many of the same results as Rickart (1950) at about the same time but did not publish his results. Answering this question amounts to showing that an isomor-

* Numbers in this form refer to the bibliography at the end of the text.
phism between semisimple Banach algebras is continuous. In 1967 Barry solved this well-known problem by using the purely algebraic Jacobson theory of irreducible representations with delicate inductive estimates in analysis and a gliding-hump argument (15). This solution stimulated research in automatic continuity, which evolved rapidly for a few years after his proof, and then developed steadily thereafter. Jointly with A.M. Sinclair he solved a conjecture of Kaplansky on the automatic continuity of derivations on semisimple Banach algebras by modifying the techniques used in the uniqueness of norm proof (18). The overall strategy is similar in both theorems. Provided there are no finite dimensional irreducible representations of the algebra, the techniques worked for purely additive derivations, yielding that the derivation had to be real linear. Subsequently simpler and alternative proofs were given of both these theorems (see Dales 1978, 2000, chapter 5). Before this breakthrough Barry published a series of results on automatic continuity, developing the theory and gaining experience with the methods. These results covered the following case: centralizers, or multipliers, of Banach algebras (6), linear operators intertwining with a pair of suitable continuous linear operators (7), operators leaving certain translation-invariant subspaces invariant (8), homomorphisms of $B(X)$ for certain Banach spaces $X$ (9) and derivations of commutative semisimple Banach algebras (14). The abstract case of intertwining operators was motivated by homomorphisms in the introduction of the paper and was a motivation for (21). Barry returned to automatic continuity after 1969 on three occasions (38, 47, 50) although by then the main thrust of automatic continuity had branched from Barry’s approach as a result of research by H.G. Dales and J. Esterle on the existence of discontinuous homomorphisms from $C(\Omega)$ (Esterle 1978a,b; Dales 1979) For a full discussion of automatic continuity and Barry’s contributions to it, see Dales (2000).

Hochschild cohomology and amenability of Banach algebras (17, 22, 24, 28, 30, 33, 36, 45, 54–56, 61–65, 67, 70, 71)

In the late 1960s cohomological ideas were starting to become important in Banach algebras and von Neumann algebras, with the few scattered results sometimes not worded in cohomological terms and with no coherent theory. Barry’s American Mathematical Society Memoir, ‘Cohomology of Banach algebras’ (28), laid the foundation of an extensive theory of (continuous) Hochschild cohomology of Banach algebras based on Hochschild cohomology for algebras (Hochschild 1945). The Hochschild cohomology theory of Banach algebras is analogous to the classical algebraic Hochschild cohomology of associative algebras with Banach modules and continuous multilinear operators in place of ring modules and standard cochains. However, many of the traditional applications of ring cohomology cannot be extended to Banach algebras without severe restrictions: examples are extensions and lifting derivations. The reasons for this are that the image of a continuous linear operator may not be closed and that a closed linear subspace of a Banach space does not in general have a closed complement. Barry emphasized that for a successful theory the topological properties of the module should match the topological properties of the algebra and the averaging required in the calculations: Banach modules should be used over Banach algebras if one is averaging over compact groups; dual Banach modules should be used if one is averaging over amenable groups, and dual normal modules should be used over von Neumann algebras. The idea that an average is just a suitable fixed-point result, which goes back to von Neumann’s construction of Haar measure on a locally compact group, is highlighted with the title to chapter 3, ‘$H^1(L^1(G),X)$ and fixed points’, in (28). Averaging in cohomology is to be interpreted in this wide sense.
Here is the application of the averaging idea in its simplest form of showing that \( \mathcal{H}^1(A, X) = 0 \) for a bimodule \( X \) over a complex unital algebra \( A \) when \( A \) contains a discrete multiplicative group \( G \) that suitably generates \( A \). This comes down to proving that for a derivation \( D \) from \( A \) to \( X \) there is an \( x \) in \( X \) so that \( D(a) = ax - xa \) for all \( a \) in \( A \); a derivation is a linear map on \( A \) satisfying \( D(ab) = aD(b) + D(a)b \) for all \( a, b \) in \( A \). Here are the constructions of \( x \) in the finite dimensional purely algebraic situation and when \( G \) is a discrete amenable group. If \( G \) is a finite group and \( A \) is the linear span of \( G \), then

\[
x = |G|^{-1} \sum_{g \in G} g^{-1}D(g),
\]

where \( |G| \) is the number of elements in \( G \). If \( G \) is an amenable group with invariant mean \( m_g \) on \( l^1(G) \), if the Banach algebra \( A \) is the closed linear span of \( G \) with \( G \) norm bounded and if \( X \) is a dual Banach \( A \)-module with predual Banach \( A \)-module \( X_* \), then

\[
\langle x, \xi \rangle = m_g \langle (g^{-1}D(g)), \xi \rangle
\]

for all \( \xi \) in \( X_* \). In cohomological terms this final result says that for these Banach algebras \( \mathcal{H}^1(A, X) = 0 \) for all dual modules \( X \) over \( A \). Barry used this as the definition of an amenable Banach algebra ((28), p. 60).

Barry studied the amenability of closed ideals, quotients and of the Banach algebra of continuous linear operators on a Banach space (28). Subsequently he modified the idea of a diagonal in the cohomology of finite dimensional algebras to an approximate diagonal and virtual continuous linear operators on a Banach space (28). Subsequently Barry modified the idea of a diagonal in Banach algebras ((28), p. 60).

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Barry studied the amenability of closed ideals, quotients and of the Banach algebra of continuous linear operators on a Banach space (28). Subsequently he modified the idea of a diagonal in the cohomology of finite dimensional algebras to an approximate diagonal and virtual diagonal in Banach algebras, and related them to amenability (30). The importance of the second cohomology group \( \mathcal{H}^2(A, X) \) is clear in the papers (28, 30, 33). By introducing the bounded group cohomology of a discrete group and showing that the second bounded cohomology group \( \mathcal{H}^2_B(\mathbb{F}_2) \) of the free group \( \mathbb{F}_2 \) on two generators is non-zero, he deduced that \( \mathcal{H}^2(l^1(\mathbb{F}_2), l^1(\mathbb{F}_2)) \) is non-zero ((28), p. 38). Bounded-group cohomology was discovered independently by M. Gromov and related to geometrical problems by him and co-workers (see Grigorchuk 1995 for relevant references and discussion). Using harmonic analysis results on the Banach algebra \( C(\mathbb{Z}_2) \hat{\otimes} C(\mathbb{Z}_2) \) and a map due to N. Varopoulos (Varopoulos 1967), Barry showed that if \( A \) is an amenable infinite dimensional abelian Banach algebra, then there is a Banach \( A \)-module \( X \) with \( \mathcal{H}^1(A, X) \) not zero (30).

While Barry was creating the cohomology theory of general Banach algebras, there was a quite independent development of norm continuous and weakly continuous cohomology theories of von Neumann algebras and \( C^* \)-algebras by Kadison & Ringrose (1971). As a result of joint consultations Barry made fundamental contributions to the cohomology of von Neumann algebras and \( C^* \)-algebras. First the fixed-point technique of (28), chapter 3, was modified to show that \( \mathcal{H}^1(N, N) = 0 \) for all von Neumann algebras \( N \) and to study the first cohomology from the Banach algebra \( l^1(G) \) into a uniformly convex Banach module over \( l^1(G) \) (22). The result that \( \mathcal{H}^1(N, N) = 0 \), or equivalently that derivations on a von Neumann algebra are inner had been proved by Sakai (1966), following Kadison (1966). Fixed-point techniques have had a role in all cases in which \( \mathcal{H}^1(N, N) \) has been shown to be zero (Sinclair & Smith 1995).

More importantly for future research, Barry was instrumental in showing that for a dual normal module \( X \) over a von Neumann algebra \( N \), \( \mathcal{H}^n(N, X) = \mathcal{H}^n(N, X) \) for all positive integers \( n \), where \( \mathcal{H}^n \) denotes norm continuous cohomology and \( \mathcal{H}^n \) denotes normal cohomology (24). In normal cohomology \( \mathcal{H}^n \) all maps are required to be multilinear normal maps, that is, are separately continuous in the weak topologies of the algebra and module. This result,
together with an important technical lemma on relative $M$-multimodular normal cohomology with respect to a hyperfinite von Neumann subalgebra $M$ of $N$, has been used in all subsequent attempts at proving $\mathcal{H}^n(N, N) = 0$ for a von Neumann algebra $N$ (see Sinclair & Smith (1995) and Christensen et al. (2003) for references).

Until 1986 the cohomology group $\mathcal{H}^n(N, N)$ was known to be zero only for $N$ a hyperfinite von Neumann algebra and for one non-hyperfinite group von Neumann algebra $M$ that Barry had constructed to satisfy ingenious relations ensuring that $\mathcal{H}^2(M, M) = 0$ (33). This single example was important in indicating that $\mathcal{H}^n(N, N)$ might be zero for a larger class of von Neumann algebras than the hyperfinite ones.

At the time that Barry was working on his memoir (28) in 1970, A.Y. Helemskii was working on a more abstract approach to the homology of Banach algebras (see Helemskii 1989). Barry wrote to Helemskii in June 1970 to ensure that they were aware of one another’s research in a letter described by Helemskii as ‘kind, generous and considerate’. Barry’s judgement on the correct level of abstraction and structure of the continuous cohomology theory of Banach algebras was excellent. He based his development on Hochschild’s algebraic version using explicit complexes of continuous multilinear maps rather than a more abstract theory based on $Ext$ and $Tor$. His approach took a middle route between the more abstract version of Helemskii (1989) and the more concrete one of Kadison & Ringrose (1971).

On the basis of Barry’s memoir these ideas on amenability were used, modified and extended by subsequent mathematicians to relate amenability in different situations to other natural good properties of the algebras. Examples of this are A. Connes showing that von Neumann amenability and injectivity are equivalent for von Neumann algebras, and that amenability implies nuclearity for $C^*$-algebras (Connes 1976, 1978), E.G. Effros studying amenability and virtual diagonals in von Neumann algebras (Effros 1988), U. Haagerup showing nuclearity implies amenability for $C^*$-algebras (Haagerup 1983), S. Popa’s results on the amenable embedding of one type $II_1$ factor in another (Popa 1994) and of the amenability of a completely positive map (Popa 1998), and Z.-J. Ruan’s results showing that the amenability of a locally compact group $G$ is equivalent to the operator amenability of the Fourier algebra $A(G)$ (Ruan 1995) and his study of the amenability of Kac (or Hopf–von Neumann) algebras (Ruan 1996).

After 1976 Barry concentrated his research in cohomology on amenability and derivation questions rather than the higher groups that were the centre of his earlier research. He showed that Fourier algebra $A(G)$ of a compact group $G$ is amenable if the set $\{d_\pi; \pi \in \hat{G}\}$ is bounded, where $\hat{G}$ is the set of equivalence classes of irreducible representations and $d_\pi$ is the dimension of the irreducible representation $\pi$ (62), Theorem 5.3). This he deduced from the calculation that the minimal norm that an approximate diagonal can have in the Fourier algebra $A(G)$ of a finite group $G$ is

$$\sum d_\pi \left/ \sum d_\pi^2 \right.$$

where both summations are over the set of all equivalence classes $\hat{G}$ of irreducible representations of $G$. This was the main motivation for Ruan to introduce operator amenability and show that the operator amenability of $A(G)$ characterizes the amenability of $G$ as mentioned above (Ruan 1995).

A Banach algebra $A$ is permanently weakly amenable, if for each positive integer $n$ each derivation from $A$ into its $n$th iterated dual $A^\ast_n$ is inner (Dales 1998). In two of his last papers Barry showed that $l^1(G)$ is permanently weakly amenable if $G$ is a free group (66) or $G$ is a...
hyperbolic group (70), completing results in Dales (1998). The proof in both cases involves ergodic properties of the action of the non-amenable group on a suitable measure space, which is the hyperbolic boundary $\partial G$ of $G$ here. The amenable group case follows from general results of Barry’s in (28).

**Perturbations in Banach algebras (39, 41, 43, 50, 52, 53, 59, 60)**

In the mid-1970s several mathematicians studied perturbations of products, representations and other structures of Banach algebras and operator algebras (see (39) and Raeburn & Taylor (1977) for references to earlier work). In late 1975 Barry Johnson (39), and independently Raeburn and Taylor (1977), proved essentially the same results on perturbations of the product, and of representations, of Banach algebras under the assumptions that certain Hochschild cohomology groups over the algebra are zero. Barry’s approach is an indication of the way he often thought out the results from the beginning in an independent way. The Raeburn–Taylor method is to prove an infinite dimensional implicit function theorem (Raeburn & Taylor 1977, Theorem 1) and show that the various spaces in the definition of the cohomology are related to derivatives of certain maps between suitable manifolds. Barry’s approach is to tackle the results directly via the Banach contraction mapping theorem and careful inductive constructions, which with hindsight is just mimicking the proof of the implicit function theorem. Here is a special case of Barry’s Theorem 2.1 (39) and Raeburn and Taylor’s Theorem 3 (Raeburn & Taylor 1977).

Let $A$ be a Banach algebra and let $\pi$ denote the usual product on $A$ as a bilinear operator from $A \times A$ to $A$. If $\mathcal{H}^2(A, A) = \mathcal{H}^3(A, A) = 0$, then there is an $\varepsilon > 0$ such that if $\rho$ is another associative multiplication on $A$ with $||\pi - \rho|| < \varepsilon$, where the norm is that of a bilinear operator, then $(A, \rho)$ is isomorphic to $(A, \pi)$ via an invertible map $T$ on $A$. Further as $||\pi - \rho||$ tends to zero, $||T - I||$ tends to zero.

The $\varepsilon$ depends on the norms of inverse maps in the cohomology calculations. Unique to Barry’s paper are a technical open mapping lemma ((39), Lemma 6.1) that enables one to weaken the hypotheses needed in some of the results in both papers (see the note added in press in Raeburn & Taylor (1977), p. 266), and several natural examples. He shows that the algebra $B(\mathcal{X})$ of all bounded linear operators on a Banach space $\mathcal{X}$ and the algebra $C(\Omega)$ of all continuous complex-valued functions on a compact metrizable space $\Omega$ have $\mathcal{H}^n(A, A) = 0$ for all positive integers $n$, and so satisfy the required hypotheses.

Barry considered a different type of perturbation result in (50, 52) on almost multiplicative linear functionals. Here is a brief description. Let $A$ and $B$ be Banach algebras and for each continuous linear operator $T$ from $A$ into $B$, let

$$T'(a, b) = T(ab) - T(a)T(b)$$

for all $a, b$ in $A$. The pair $(A, B)$ is said to be an AMNM pair, *Almost Multiplicative maps are Near Multiplicative*, if for each positive $\varepsilon$ and $K$ there is a positive $\delta$ such that if $T$ is a continuous linear operator from $A$ into $B$ with $||T|| < K$ and $||T'|| < \delta$, then there is a multiplicative linear map $T'$ from $A$ into $B$ with $||T - T'|| < \varepsilon$. The main result of (52) is that if $A$ is an amenable Banach algebra and $B$ is a Banach algebra such that as a Banach $B$-module $B$ is isomorphic to $(B, _*)'$, where $(B, _*)$ is a Banach $B$-module, then $(A, B)$ is an AMNM pair. Other results of this type, examples and counter-examples are studied in (50), (52) and (59).
Results on derivations run throughout Barry’s research from 1969 (14) until his last research (69). The derivation results that involve automatic continuity are discussed in that section and those that are close to general cohomological problems are discussed there. Here are some others.

In 1972, jointly with S.K. Parrott, Barry considered two closely related problems associated with a von Neumann algebra $N$ on a Hilbert space $H$ (29). Let $K(H)$ denote the algebra of compact linear operators on $H$. Is a derivation from $N$ into $K(H)$ inner? If $b$ is a bounded linear operator on $H$ and $xb - bx$ is in $K(H)$ for all $x$ in $N$, is $b$ in $N' + K(H)$, where $N'$ is the commutant of $N$? They answer ‘yes’ to both questions for von Neumann algebras $N$ that do not ‘contain certain intractable type II$_1$ factors as direct summands’. They solve the problem first for commutative von Neumann algebras and then reduce their other cases to this. The full problem was solved by Popa (1987) (see also Popa & Rădulescu 1988; Rădulescu 1991).

One of the intriguing problems that Barry solved in 2000 concerned ‘local derivations’ from a $C^*$-algebra $A$ into a Banach $A$-module $X$. He showed that if $T$ is a continuous linear operator from $A$ into $X$ such that for each $a$ in $A$ there is a derivation $D_a$ from $A$ into $X$ with $T(a) = D_a(a)$, then $T$ is a derivation from $A$ into $X$ (68). The hard step of the proof involves the particular case $A$ equal to $C_0(\mathbb{R})$ and $X$ equal to the dual of the Banach module $C_0(\mathbb{R}) \otimes C_0(\mathbb{R})$. Kadison (1990) had proved this result for $A$ a von Neumann algebra and $X$ a dual module over it.

If $D$ is a derivation on the group algebra $L^1(G)$ of a locally compact group $G$, is there a bounded regular measure $\mu$ on $G$ such that $D(x) = x*\mu - \mu*x$ for all $x$ in $L^1(G)$? This was one of the problems that motivated Barry to look at the amenability of Banach algebras. Barry returned to this problem in 2001, having answered ‘yes’ for amenable, SIN and some matrix groups in (28), Proposition 4.1. Using detailed properties of connected Lie groups he showed that the answer is yes for $G$ a connected locally compact group (69). In (56) Barry showed that if $G$ is a locally compact group, then each derivation from $L^1(G)$ into $L^\infty(G)$ is inner. The proof is interesting in that the usual linear averaging argument used to show derivations are inner is replaced by a supremum of a suitable set ((56), Lemma), so by a nonlinear process.

Centralizers and other research (1–3, 5, 10–13, 16, 19, 26, 27, 31, 32, 37, 44, 46, 47, 57, 58, 66)

Barry’s PhD thesis and several early papers were on centralizers of topological algebras (1, 2, 10) with a detailed study of the complications that can occur in algebras whose left and right structures are rather different. Centralizers are now called multipliers because of their connections with harmonic analysis. They have also been helpful in understanding the structure of non-unital $C^*$-algebras. Throughout his career he returned at intervals to problems associated with the Banach algebra $L^1(G)$ and its associated multiplier algebra $M(G)$ of bounded Borel measures on the locally compact group $G$ (3, 12, 13, 47, 66).

At the time when there seemed a faint possibility of a hierarchy of $*$-algebras $C^*$, $AW^*$ and $W^*$ (= von Neumann) algebras, a class of algebras called $QW^*$ algebras was introduced in 1965 by G.A. Reid. These contained the $W^*$-algebras and were contained in $AW^*$-algebras. In 1967 Barry showed that $QW^*$-algebras were $AW^*$-algebras (11).

He and Simon Wassermann gave an example of the failure of the slice map criterion in $C^*$-algebras (44).

His remaining research was scattered over different areas of functional analysis and Banach algebra theory.
**Barry Edward Johnson**

**General impact of Barry Johnson’s research**

The importance of Barry’s research can be seen by looking at the remarks, contents and list of references of the following lecture notes and books on Banach algebras, automatic continuity and Hochschild cohomology of various kinds of Banach algebras: Bonsall & Duncan (1973), Sinclair (1976), Hellemskii (1989), Palmer (1994, 2001), Sinclair & Smith (1995) and Dales (2000). These books indirectly contain a detailed assessment of some of Barry’s research.

**ACKNOWLEDGEMENTS**

George Willis and I acknowledge our deep indebtedness to Barry Johnson for stimulating and challenging supervisions, and for turning us from dubious foreign PhD students into research mathematicians. For each of us, being supervised by Barry was a turning point in our mathematical career and the luckiest academic thing that happened to us.

I should like to thank the following people for help, information, and checking this memoir: Margaret Johnson and Barry’s family, John Ringrose, Erik Christensen, Fereidoun Ghahramani, Zhong-Jin Ruan, Roger Smith and George Willis.

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**REFERENCES TO OTHER AUTHORS**


BIBLIOGRAPHY

These papers are listed in the order in Barry Johnson’s last CV with the addition of (42), which was omitted from his CV, (70), which was found as a complete unchecked typescript in his papers, and (71), which was in progress when he died.

(20) Continuity of homomorphisms of algebras of operators II. J. Lond. Math. Soc. (2) 1, 81–84.
(33) 1974 A class of \( II \), factors without property \( P \) but with zero second cohomology. Ark. Mat. 12 (2), 53–159.


(70) — (With M.C. White) A non-weakly amenable augmentation ideal. (Manuscript in preparation.)

(71) — Derivations into iterated duals of group algebras and similar modules. (Manuscript found in Barry’s files; to be submitted.)

(72) — (With F. Gourdeau & M.C. White) The cyclic and simplicial cohomology of $\ell^1(N)$. (Manuscript in preparation.)