BIOGRAPHICAL
MEMOIRS

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S. M. Rees

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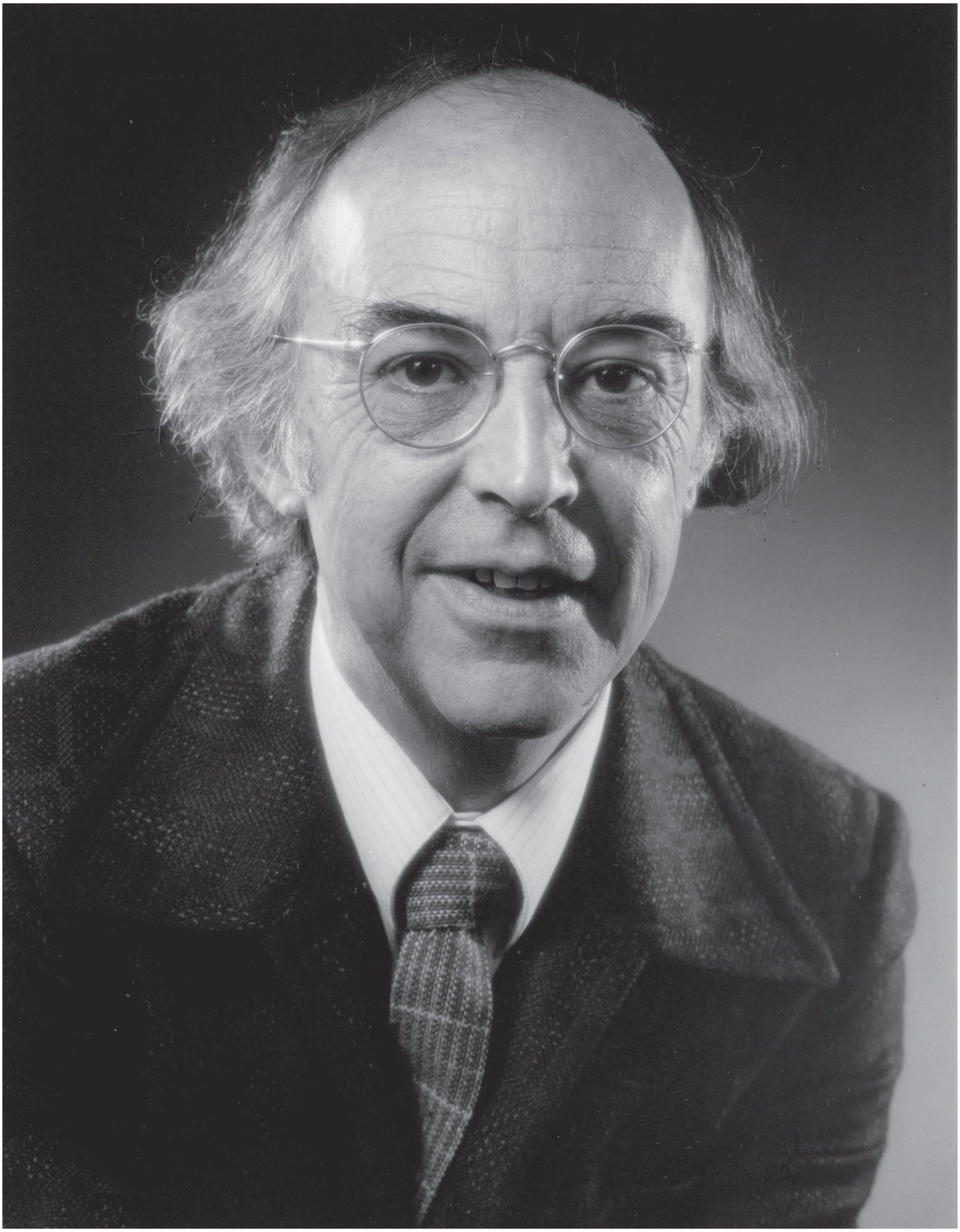

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Elected FRS 1984

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Bill Parry was born in Coventry in 1934 and died in Coventry in 2006. In between these dates was a life that developed in an unpredictable fashion. Out of inauspicious beginnings, by dint of exceptional personal qualities as well as intellect, Bill forged links with the new and foreign field of ergodic theory, became a leading exponent, and was the prime mover in establishing the field in the UK.

The early years

William ('Bill') Parry was born on 3 July 1934 in Coventry, the second youngest of the seven children, four boys and three girls, of Richard and Violet Irene Parry (*née* Sewell). Richard Parry was of Welsh ancestry, born in Manchester and orphaned at an early age. A sheet metal worker by trade, and a devoted trade unionist, he moved to Coventry in the 1930s in search of employment. Bill's mother was born in Wymondham, near Norwich. Like Richard, she was politically active, as a young woman supporting the suffragette cause, and later on the Communist Party in the 1940s. So they were a family without material and social privileges, but bright, active and rebellious, with politics looming large: Bill's elder brother Eddie was a trade union activist and a member of the Communist Party, one of his sisters was a supporter, and Bill himself joined the Communist Party at the age of 18 years.

The start of Bill's formal schooling coincided with the outbreak of war, with his primary school education at Whoberley Infants and Juniors interrupted by 18 months' evacuation to Oswaldtwistle in Lancashire, starting the day before the Blitz. Earlier bombardment of Coventry is remembered in one of the poems he wrote in retirement, 'Air raid'. He describes a family's pragmatic approach to the danger: children 'stowed' under the table for protection,

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where they could comfort each other, the father watching the bombing from the back door, with Bill beside him, where he felt supported.

Bill's education, like many other things about him, was unusual. In 1945 he failed his 11+ examination, as did most of his neighbourhood peer group. Two years later he passed the 13+ to Coventry Technical Secondary, which promised training in technical skills, none of which he acquired. By this time, however, he had developed an interest in mathematics, inspired by a mathematics teacher at his school whose identity we do not know, who went by the name 'Pedro'. At the age of 17 years Bill took his higher school certificate, but passed only in pure mathematics. So he started work, on milk rounds and in petrol stations, while continuing his education in evening classes in Coventry and Birmingham. At 19 years of age he obtained his Higher Schools Certificate in Pure and Applied Mathematics and Physics, and applied to go to university. Hyman Kestelman and Theodor Estermann at University College London interviewed him and, recognizing a considerable talent despite inadequacies in his grounding, made the decision that resulted in the award of a state scholarship. Kestelman, in particular, was regarded by Bill as a profound influence.

Having arrived in London, Bill took full advantage of the cultural opportunities available, while also engaging in political activities. He played poker, but with no more success than in his sales of *The Daily Worker*, or so he claimed. His ambition was to do his doctorate at Cambridge, but because he missed a first-class honours degree he went to Liverpool to study for an MSc, and wrote a dissertation on measure theory, formally under the supervision of the analyst Bill Newns, of whom he later spoke warmly to the present author (who was a colleague of Newns's in the mid 1980s). Bill was also much occupied by politics at this time. Oppressed by the crushing of the Hungarian uprising, he left the Communist Party and was persuaded by a formidably self-educated dock worker and activist, Bill Hunter, to join a Trotskyist group. In 1957, Bill left Liverpool to do his PhD at Imperial College, London, with Yael Naim Dowker.

Yael and Benita

This was an intriguing choice, one of many examples of Bill's knack of making unexpected intellectual links with people. In this case the link was indirect, involving three people of diverse backgrounds and adventurous academic inclination, for it was Jal Choksi who suggested that Bill apply to work with Yael. Jal Choksi, originally from Mumbai, was a research assistant working for Professor Geoffrey Walker FRS, the head of department at Liverpool. Jal was finishing his thesis on measure theory and had recently arrived from Manchester, where he had been a PhD student of Harry Reuter's. He was just two years older than Bill. They soon became friends, spending time together and sharing an office—along with all the other postgraduates and postdoctoral researchers. Bill soon grasped the point of Jal's thesis, even generalizing a part of it, as well as related work by T. P. Srinivasen, as part of his MSc dissertation. However, Jal advised Bill against continuing to a PhD in this type of measure theory, suggesting instead that he study ergodic theory—as he planned to do himself—and that Yael Dowker was the unique choice of supervisor in ergodic theory in the UK at this time.

Yael was an Israeli who moved to the UK with her husband in 1950 to avoid the attentions of the House Un-American Activities Committee, after spending nearly 10 years in the USA. She studied for her MA (with O. Zariski) at Johns Hopkins University, where she met and married the Canadian topologist Hugh Clifford Dowker. After the war she did her PhD, officially at Harvard, but her *de facto* advisor was W. Hurewicz, a professor at Massachusetts Institute of Technology. Hurewicz's main research interests were, like Dowker's, in topology, and particularly in dimension theory, but he was also interested in ergodic theory. It was in this area that he advised Yael. Ergodic theory was not a major subject of study at this time, but Hurewicz directed Yael to the literature, the classic works of G. D. Birkhoff, and the emerging work of the Russian masters such as A. N. Kolmogorov (ForMemRS 1964). Hurewicz himself proved one of the main ergodic theorems, in the absence of an invariant probability measure (Hurewicz 1944). Yael briefly had a fellowship at the Institute for Advanced Study, in the name of Emmy Noether, before the move to Britain, which was tough, especially for Hugh. However, Yael obtained a position at Manchester, in the vibrant department led by M. H. A. Newman FRS, and then Hugh found a post at Birkbeck College in London. Subsequently, Yael moved to Imperial College. She was possibly the only person in the country working in ergodic theory. There were one or two other people doing work in dynamics, for example Mary Cartwright FRS, but there were few personal links. Yael, like most others at the time, got her impetus for her work from the literature. Her papers from the 1950s are in classical ergodic theory and some dynamics. She wrote one paper with Paul Erdős, whom she knew well. Her first personal contact with researchers in Russia came during a six-month visit to Moscow in 1959—a visit that at the time was unusual, and also turned out to be very significant, because Yael attended seminars given by L. M. Abramov, V. A. Rohlin and Y. G. Sinai (among others) and the topics included Kolmogorov's new theory of entropy.

Yael was surprised, but pleased, to receive an enquiry from Bill Parry about studying for a PhD. She knew nothing about him but he looked an interesting prospect. His MSc at Liverpool, unusually, included some original research. However, because Bill did not have a first-class honours degree, the transfer to PhD study was not straightforward. Imperial had six PhD studentships available, but the head of department at Imperial had six first-class students lined up for them. Yael's instinct was that Bill deserved a chance. She sought the support of her young colleague Walter Hayman FRS, who was sympathetic and tried to help, but to no avail. So Yael took the train to Liverpool to discuss the matter with Walker, who agreed to hand over one of the Liverpool PhD studentships—of which there were at most two—for Bill to take to Imperial to study with Yael. Two years later, well into his PhD, Bill was awarded an Arthur Jubber studentship of the University of London, but Yael Dowker's initiative, and Walker's generosity, gave him the start he needed. Thus began a lifelong friendship between Yael and Bill that was to be important in both personal and mathematical terms. On both counts, with a 12-year age difference, the relationship was essentially that of an older and younger sibling, with the dynamics naturally changing over time.

Having returned to study in London, Bill met the other woman who was to be of lifelong importance to him: his wife, Benita Teper. The child of immigrants from Lithuania and Poland, she was born in South Africa, and, in a country awash with oppositional politics, had been a member of a small radical group. Unlike Yael, Benita came to Britain on her own. However, many of her compatriots were on the move, and on arrival she shared a flat in London with South African friends of similar political affiliations, where Bill was soon to join the household. Before going to the London School of Economics to study history, she did various routine jobs to help sustain the very meagre existence they shared.

Bill and Benita met on the 1958 Aldermaston march. Bill was selling the publications of a Trotskyist group that developed in 1959 into the Socialist Labour League (SLL), led by Gerry

Healey. Meetings of the nascent SLL fostered the relationship between Bill and Benita and precipitated their marriage (in 1958) because, to their amusement, some senior members of the League frowned on cohabitation. In 1960 Bill obtained his doctorate and moved to his first teaching post in Birmingham. He was also offered a post at Queen Mary College, London, but he and Benita chose Birmingham because of a desire to escape from the influence of the League. Bill ceased to be a member of the SLL in 1960, and Benita left about a year later, although both remained committed socialists and shared an implacable opposition to capitalism as a calculated but irrational system engendering injustice and inequality.

Laying the groundwork

Some of Bill's long-term research interests emerge in his first papers. His first published paper on β transformations (1)* was the subject of his thesis. This paper built on results of A. Rényi, who reviewed the paper for *Mathematical Reviews* (Rényi 1963). (The Hungarian mathematician Rényi was one of the most eminent ergodic theorists of the day. Yael Dowker was interested in his work, and unusually for a mathematician from eastern Europe, Rényi visited London, and Dowker, once or twice. It is not certain that Rényi and Bill Parry ever met.) Rényi had shown ergodicity of the β transformation

$$T_\beta : x \mapsto \beta x \bmod 1 : [0,1) \to [0,1)$$

for $\beta > 1$, with respect to Lebesgue measure, and the existence of an absolutely continuous measure ν_β with respect to Lebesgue measure. Bill gave an explicit expression for the measure.

As Rényi points out in his article, this description of the measure was found at the same time, and independently, by A. O. Gelfond. It was probably also worked out independently by Yael Dowker during her visit to the USSR in 1959. However, in his paper, Bill also studied β-expansions of real numbers and established connections between these and the dynamics of the associated transformation, which laid the groundwork for engrossing studies that continue to this day. Define the integer $b(x)$, which is $\geqslant 0$ and $\leqslant \beta$, by the equation

$$\beta x = T_\beta(x) + b(x).$$

As usual in dynamics, T_β^n denotes the n-fold decomposition of T_β. If we define $b_k(x) = b(T_\beta^k(x))$, then $\eta_\beta(x) = (b_k(x))$ is a sequence of integers such that

$$b_k(T_\beta(x)) = b_{k+1}(x)$$

and x has the β-expansion

$$x = \sum_{k=1}^{\infty} b_k(x)\beta^{-k}.$$

This is an example of symbolic dynamics. The closed set

$$V_\beta = \overline{\{(b_k(x)) : x \in [0,1)\}}$$

* Numbers in this form refer to the bibliography at the end of the text.

is invariant under the shift that sends the sequence (b_k) to the sequence (b_{k+1}). There is a simple description of which sequences are in the set V_β. Define $\eta_\beta(1)$ in the same way as $\eta_\beta(x)$ for $x \in [0,1]$. Then V_β is the set of all sequences $\leqslant \eta_\beta(1)$ in the dictionary order. Bill noted that V_β is a subshift of finite type if and only if $\eta_\beta(1)$ is eventually mapped to 0, and that V_β is sofic if and only if $\eta_\beta(1)$ is eventually periodic. The β for which these happen have come to be known, respectively, as *simple β numbers* and *β numbers*. Bill showed that simple β numbers are necessarily algebraic, with all conjugates of modulus $\leqslant 2$. B. Solomyak improved this (Solomyak 1994) to modulus $\leqslant (1+\sqrt{5})/2$. This bound is sharp. One related problem that has generated great interest over the years is the nature of the set of periodic points of T_β. Bill himself returned to this particular problem in about 1980, contributing to a paper by his colleague Klaus Schmidt (Schmidt 1980).

Subshifts of finite type were to assume great importance in the study of dynamics in the following years, as hyperbolic dynamical systems became increasingly prominent examples. A subshift of finite type is a topological analogue of a Markov chain: a type two shift, for example, is a space of sequences with entries in a finite set for which every successive pair of elements is in an allowable set of pairs. Bill was one of the first to identify the importance of the subshifts of finite type, in a paper where he called them *intrinsic Markov chains* (4). This paper, among other things, identified the unique measure of maximal entropy, always a Markov measure for a subshift of finite type and now often known as a *Parry measure*.

Coming together

Postdoctoral research opportunities in the 1960s were fairly limited, but during his time at Birmingham, initially through Yael Dowker's connections, Bill had several extended visits to Yale, as a visiting lecturer for the academic year of 1962/63, as a visiting researcher in the summer of 1964, and in the autumn of 1966, after he had moved to Sussex. He wrote one paper with S. Kakutani (2), which gave a family of examples. For each integer k a σ-finite-measure-preserving transformation T is given such that the k-fold product of T with itself is ergodic, but the $k+1$-fold product is not. This is in contrast with a result of Paul Halmos for a finite-measure-preserving transformation T that says that if $T \times T$ is ergodic, then the same holds for the k-fold product. (This is because, for a finite-measure-preserving transformation T, ergodicity of $T \times T$ is one of the equivalent statements of T being weak-mixing: another is that $T \times S$ is ergodic for any ergodic S.) The examples that Bill used were random walks from a paper by J. Gillis (Gillis 1956). This illustrates another strand of his research for many years, in abstract ergodic theory, as does his study of Hurewicz's ergodic theorem (5), and his version of McMillan's Information Theorem without an invariant probability measure (3).

Measure-theoretic entropy was a major preoccupation of the period, and was an important topic in Bill's doctoral studies. Entropy was a new invariant of a measurable dynamical system, introduced by Kolmogorov (1958) to distinguish between two probability-measure-preserving transformations with the same continuous spectrum. It was this theory that Bill learnt from Yael during his doctoral studies, news from the seminars in Moscow passed on in London, perhaps the first communication of the new theory in the West. Yael Dowker was responsible for translating some of the Russian papers, by Rohlin and Sinai, for example, for the London Mathematical Society. These translations appeared from 1960 onwards. It was known, from work of Halmos, that probability-measure-preserving transformations with purely discrete spectrum were determined up to measure isomorphism by the spectrum. Bill looked at a generalization of discrete spectrum that he called *quasi-discrete spectrum*. As with discrete spectrum, Bill showed that quasi-discrete spectrum was essentially a complete invariant and induced a structure of a continuous transformation of a compact topological space. In fact a system with quasi-discrete spectrum turns out to be measure-isomorphic to anaffine transformation of a compact Abelian group with Haar measure. Bill wrote at least five papers on the subject, two (6, 13) with the established American ergodic theorist Frank Hahn, and three more (7–9) with his group theory colleague from Birmingham, A. H. M. Hoare. A number of ergodic theoretic properties of such systems are proved in these papers, such as countable Lebesgue spectrum and conditions for ergodicity in terms of ergodicity of quotients.

Research interest in skew products converged from several different directions in the early 1960s. The increasing trend of studying topological and measure-theoretic structure together, which is evident in Bill's work, was also in evidence in research in other countries. An important outcome was Furstenberg's study of distal transformation groups, and his distal structure theorem (Furstenberg 1963), which shows how dynamical systems with the purely dynamical property of distality are built from a tower of skew-product extensions.

In Birmingham, Bill acquired his first graduate students. The very first was Dan Newton. Bill and Dan worked together on Gaussian processes, and factors of Gaussian processes, with special interest in entropy and Lebesgue spectrum. Their joint paper (10) examined the entropy and spectrum type of these dynamical systems, instituting a study that was of great interest to ergodic theorists later on, such as J.-P. Thouvenot, M. Lemanczyk and F. Parreau. Dan encouraged Peter Walters, a friend in the year below him and, like Dan, from Derbyshire, to do his doctorate with Bill, one of the youngest lecturers in the department and doing the most interesting mathematics. Peter, and Mohamed Haque from Pakistan, both started work with Bill in the autumn of 1964. In January 1965 Bill's job moved to Sussex, where he was appointed a senior lecturer. His three students moved *en bloc*. For a few months Bill commuted from Birmingham while he and Benita were househunting. In the academic year 1965/66, Yael Dowker organized a small working seminar on ergodic theory at Imperial College. Several of the papers read, perhaps most, were in Russian, but the group included two capable translators in Yael Dowker and Dan Newton. The London participants included the probabilist Harry Reuter and Yael's colleague Cyril Offord FRS. The Sussex group made good use of the journey from Lewes station—where they always stopped for tea—and talked mathematics throughout the journey.

Measure-theoretic entropy was certainly a major topic in this seminar. Bill wrote some fundamental papers on the existence of generating partitions (11) and conditions for partitions to be generating, in which the entropy is relevant (12). Given a measure-preserving transformation T of a space X, a partition ξ of X is a *generating partition* if the partition $\bigvee_{n=0}^{\infty} T^{-n}\,\xi$ is the partition into points. The result (11) was obtained independently in the USSR by Rohlin, whereas (12) was published in Russian in the mathematics section of *Doklady Akademii Nauk SSSR*, after Bill sent it to Sinai. (It appeared in the English translation of the journal under the name of V. Perri.) In the summer of 1966, Bill attended the International Congress of Mathematicians in Moscow and met some of the Russian ergodic theorists for the first time: Sinai, A, M. Stepin and A. B. Katok.

In the autumn of 1966, Bill again visited Yale, and there he gave a course of lectures on entropy. A few years later the lectures were published in book form (14). In the introduction, Bill explains that his intention is to develop the abstract theory and give some applications, regretting that he has had to omit results of Sinai, D. V. Anosov and M. S. Pinsker, and, not surprisingly, given his work on generators, foresees the arrival, just months later, of Ornstein's celebrated proof of the completeness of the invariant for Bernoulli systems (Ornstein 1970).

The golden years

In 1967 Bill and Benita's daughter, Rachel, was born. In 1968 Bill made his final move, to Warwick, where he became a professor in 1970 and remained until his retirement in 1999. The University of Warwick had been founded in 1964; under the dynamic and inspiring leadership of Christopher Zeeman (FRS 1975), the Mathematics Institute quickly became one of the most exciting places to work in the country, and was a magnet for leading researchers throughout the world. Zeeman persuaded the Science Research Council (SRC) to fund annual year-long Symposia in Mathematics (Zeeman 2004). These symposia are still an institution at Warwick, although funding is applied for individually, for each programme. The events are smaller now, with activities concentrated in a few weeks, mostly in the summer. In the 1960s and into the 1970s, visitors came for up to a year at a time. In the academic year 1968/69, Larry Markus and Christopher Zeeman organized the Symposium in Differential Equations and Dynamical Systems. In 1967, Stephen Smale had introduced his programme of the study of dynamical systems, in particular hyperbolic dynamical systems, or *Axiom A systems* as he called them (Smale 1967). Two young American researchers, Rufus Bowen and John Guckenheimer, came to Warwick for 18 months. Stephen Smale, Mike Shub, Charles Pugh, Mo Hirsch, Jaco Palis, Bob Williams, Nancy Koppel, Anatole Beck, Benjy Weiss, Joe Auslander and Roy Adler all came to the symposium for extensive periods. Zeeman invited Peter Walters, who was halfway through an instructorship in Berkeley, to come to Warwick, not only for the symposium but also for a three-year SRC postdoctoral fellowship. Interaction between researchers in smooth dynamical systems and ergodic theorists was increasing dramatically. The minority interest of ergodic theory had become not only mainstream but centre-stage. Bowen, who, perhaps more than anyone, brought measure-theoretic and ergodic-theoretic properties of smooth dynamical systems to the fore, died tragically early at the age of 31 years, just 10 years later.

In the spirit of the era, Bill's interests in ergodic and topological dynamical systems came yet closer. He gave some constructions, such as time-changing a flow to produce weak mixing (21). He had a particular interest in nilmanifolds, including the dynamics of affine transformations of nilmanifolds, and in nilflows (15, 17). A nilflow is the left action of one parameter subgroup on a quotient $N/\Gamma = \{n\Gamma : n \in N\}$ for a nilpotent group N and discrete subgroup Γ, usually with N/Γ compact. Bill showed that several dynamical properties, such as ergodicity, minimality, or entropy of a particular value, were inherited from the same property for a maximal torus factor. (It was mainly this work of Bill Parry's that the present author studied, rather late in the day, when she began studying with Bill in 1976.) There was also an important example with Peter Walters (18), giving an example of a minimal distal homeomorphism of an infinite-dimensional torus that is not coalescent, meaning that its centralizer includes continuous maps that are not homeomorphisms. In (16) and (20), several Baire category results on weak mixing and ergodicity are proved for skew products.

In the spring of 1970 Bill met the emerging community of French ergodic theorists when he talked at the Laboratoire de Probabilité at the Sorbonne. His subject was nilflows. François Ledrappier remembers the immediate attraction: the talk was fun. Bill's invited talk in the Ordinary Differential Equations and Dynamical Systems section of the Nice International

Congress of Mathematicians in 1970 (19) was also devoted to his interest in these algebraic examples. He then went to the USA, spending the autumn of 1970 at the University of Maryland, and the rest of the academic year at the University of California in Berkeley.

Problems

During the hedonistic 1960s and early 1970s Bill and Benita caught up on the good times they had missed—rock music and parties, cuisine, long hair and outré clothes. It was not to last, even though they did not return to the austerities of their early years together. In the mid 1970s, Bill began to experience episodes of petit mal, instances of complete absence of consciousness. This was troubling for a mathematician, and necessitated dissimulation for a teacher. Bill concealed 'blank-outs' from his students, affecting to be simply thinking in front of the blackboard. In December 1976, he went to London for tests, one of which induced a full epileptic seizure, and, overall, left Bill in a state of debilitation from which he took some months to recover. For the rest of his life he took powerful drugs, which were constantly under review. As medical advances were made, the medication regime became more sophisticated. Keeping the condition stabilized, however, took a heavy personal toll, yet, as always with Bill, what projected was not the private anguish, but receptivity to the thoughts and ideas of others.

At this time, Bill's interest in subshifts of finite type intensified. The impetus was an important paper by R. F. Williams (Williams 1973) and an algebraic condition that he called shift equivalence, which was shown to be equivalent to topological conjugacy. Williams also formulated a stronger condition called strong shift equivalence, which he initially thought was the same as shift equivalence. The original paper in *Annals of Mathematics* gave a 'proof' to this effect. The erratum in the next volume pointed out that this was not the case. Bill was fascinated by the problem, and thought about it for a number of years. (The problem was finally resolved in the late 1990s (Kim & Roush 1999).) Another indicator of Bill's increasing interest in subshifts of finite type is a paper with Dennis Sullivan (22), which considered flow equivalence between suspension flows of subshifts of finite type and gave a criterion in terms of shift equivalence. In (23) Bill Parry and Bob Williams revisited their problem, which they were again unable to solve, although they found another concept similar to shift equivalence, which was, again, equivalent to topological conjugacy. Bill had yet another go later (29).

Bob Williams's original paper was probably motivated in part by Ornstein's result (Ornstein 1970) about metric entropy's being a complete invariant for Bernoulli shifts. It is not true that topological entropy is a complete invariant for subshifts of finite type. However, Bill obtained a type of topological equivalence for subshifts of finite type, called *finite equivalence*, for which topological entropy was a complete invariant (24). The same result was found independently by R. Adler and B. Marcus. There was much general interest in finding extensions and analogues of Ornstein's result. The best known extension is probably Keane & Smorodinsky's result (Keane & Smorodinsky 1979) that Bernoulli shifts with the same entropy are *finitarily isomorphic*; that is, isomorphic via an invertible map that is continuous almost everywhere, as is its inverse. This does not mean that the isomorphism is equal almost everywhere to a homeomorphism, a concept that Bill and his student Selim Tuncel (now a professor and chair of department in Seattle) referred to in their book *Classification problems in ergodic theory* (31) as *block-code homomorphism*. (The book that is probably Bill's best known, *Topics in ergodic theory* (30), was published about a year earlier, in 1981.) Bill investigated several possible further strengthenings of finitary equivalence. In (26) he gave an example of two subshifts that are finitarily equivalent but not equivalent by an isomorphism with finite expected code length. He did this by showing that finitary equivalence implied that associated information functions were cohomologous. This was refined in (27). In (28) Bill and Selim attempted to find a new complete invariant of shift equivalence of Markov chains, an analytic function called β, from which all known invariants could be computed. They were unable to solve the problem, although their conjectures were supported by many examples.

The birth of orbit counting

An important new strand in Bill's work, and possibly his best work, appeared in his papers with his student Mark Pollicott in the early 1980s on numbers of closed orbits of Axiom A flows, a subject that was to be a major theme in Pollicott's work. Interest in thermodynamic formalism was growing, with the work of D. Ruelle, and its introduction as a technique in dynamical systems by Rufus Bowen. By the time of his death in 1978, Bowen had already established the essential topological and measure-theoretical equivalence of Axiom A systems to subshifts of finite type, and of Axiom A flows to suspensions of subshifts of finite type. In 1982, Pollicott, who was an MSc student at Warwick, approached Bill to ask him whether he would supervise him for his PhD. Bill agreed, and set Mark the task of reading Ruelle's book on thermodynamic formalism (Ruelle 1978) for his MSc dissertation. Before the end of the summer, Bill had assigned Mark his PhD project: to obtain the expected asymptotic formula for the number of periodic orbits of an Axiom A flow of period $< T$, using Bowen's development of symbolic dynamics for Axiom A flows, and Ruelle's thermodynamic formalism. This formula had been conjectured by Bowen and had been proved in the case of geodesic flows on manifolds of constant negative curvature by Margulis. By the middle of the autumn of 1982, Mark had completed the proof, modulo a theorem stated as an exercise in Ruelle's book, of which the proof was established in correspondence with Ruelle.

The paper (32) was submitted to *Annals of Mathematics* in the spring of 1983, and was published in November. The proof used the subshift of finite type coming from the symbolic dynamics, and an analysis of the Ruelle zeta function, which is expressed in terms of thermodynamic formalism, to obtain the formula. This paper, which combined techniques from dynamics and analytic number theory, signalled the birth of a new area of dynamics that is shared with geometry, number theory and theoretical physics. Mark Pollicott is recognized as the leading expert on thermodynamic formalism on the world stage. This continues to be a hugely important tool in computation problems on the growth of closed orbits in dynamics, and of closed geodesics in geometry. Bill collaborated in a further paper on the subject (33), and also in a book (34). The origin of this book was a lecture course at Warwick in 1984, which was projected to be given by Bill but was, in the event, given at the first delivery by Mark, because Bill had pneumonia. Bill talked about the work at the Rufus Bowen memorial lectures that he delivered in Berkeley in 1985. Bill's student Richard Sharp (PhD 1990) was also initiated into the theory of orbit counting in dynamics. His first paper was an analogue of Merten's theorem in number theory (Sharp 1991). His interest in the subject, too, continues to this day.

In 1984, Bill was elected a Fellow of the Royal Society, fittingly at this time of highly successful convergence of his favourite ideas in mathematical analysis. In the same year, Bill became chair of the Mathematics Institute, a post he held until 1986. Despite having, in the past, threatened resignation if he ever had to take this post, he was a very successful chair: sympathetic, approachable, but also politically astute.

Full cycle

Bill Parry's interest in the larger class of skew product extensions re-emerged in the 1990s, when the subject again became very topical. Bill's own interest had never died down completely. A study by him and Henry Helson (25) of spectral properties of skew products was adapted (Mathew & Nadkarni 1984) to produce an example with spectrum of finite Lebesgue multiplicity. Parry and Helson had proved genericity statements about Lebesgue spectrum for a set of $\mathbb{Z}_2$-skew-product extensions of ergodic measure-preserving transformations in general. Mathew and Nadkarni specialized to what they call *von Neumann transformations*. They showed that the Lebesgue spectrum component could be of multiplicity two. Other finite multiplicities were also shown to be possible.

The general interest in skew products in the 1990s was nurtured by the search for a manageable enlargement of the class of hyperbolic systems. For a hyperbolic map there are two transverse foliations, invariant under the map, with leaves of one foliation expanded out exponentially by iterates of the map, and leaves of the other contracted exponentially. The interplay between the foliations leads to stability, essential isomorphism to a subshift of finite type, isomorphism to a Bernoulli shift (with respect to a natural class of invariant measures), and many other properties. The foliations do not figure in the definition of a hyperbolic dynamical system, which is expressed in terms of an invariant splitting of the tangent bundle.

For *partially hyperbolic* dynamical systems, the splitting includes a neutral direction. For such systems there is still a substantial theory. Skew products, with a hyperbolic dynamical system as base, form an important class of examples. It is reasonable to expect stability results of some sort. This led to the formulation of the concept of *stable ergodicity*. A smooth volume-preserving skew-product map f is *stably ergodic* if all volume-preserving skew-product maps in some neighbourhood of f in a suitable topology, such as C^2, are ergodic. One early result of this form was due to R. Adler, B. Kitchens and M. Shub (Adler *et al*. 1996) for torus extensions. It was generalized by Parry and Field (37) to compact Lie groups, using the case of Livšic's theorem proved by Parry and Pollicott (36). Livšic's theorem is about *coboundaries*. In this context, having fixed a map T, a cocycle is a function f of the form $g \circ T - g$. A basic version of Livšic's theorem says that if f is Lipschitz, T is Axiom A, and g is measurable, then g is Lipschitz. Bill's student Charles Walkden was introduced to the subjects of skew products, stable ergodicity and Livšic's theorem. They remain important subjects in his research (Walkden 1999).

In another paper, Bill initiated a study of skew products (35) that turned out to be isomorphic to 1-sided Bernoulli shifts, this illustrating an important distinction between the isomorphism theory for 1-sided and 2-sided Bernoulli shifts. He considered a class of skew products with 1-sided Bernoulli shift as base, and described a set of skew products that were measure isomorphic to the base. This was later generalized (Hoffman & Rudolph 2002). Their *Annals of Mathematics* paper is concerned with giving a condition for measure-isomorphism to a 1-sided Bernoulli shift, analogous to the very weak Bernoulli condition for 2-sided shifts obtained by Ornstein and Weiss. They then checked this condition for a large class of skew products that they called the $[T, \mathrm{Id}]$ examples, for an ergodic transformation T. In Bill's case, the ergodic transformation T was an irrational rotation.

In 1999 Bill reached formal retirement age. Meanwhile, Benita, now past the usual retirement age in British universities, began her academic career when the work she had been producing in postcolonial studies since the 1970s while unattached to any institution, and supported in multiple ways by Bill, was belatedly recognized. In 1998 she was appointed to an honorary chair at Warwick, then a part-time chair in 2005, and, in 2006 (unhappily, during Bill's last illness), she received an honorary doctorate from the University of York. For Bill, there was no abrupt transition or sudden disconnection with the workplace. There was a garden to care for, a cat commanding attention (as always), and Rachel's family, including his two granddaughters, to visit in North Wales. However, he continued writing papers, especially with Mark Pollicott, who moved from the Fielden chair at Manchester to a chair at Warwick in 2004. Bill's working pattern had always included extensive periods at his home in the village of Marton, in Warwickshire. Probably all his graduate students found this easy to deal with, as his appearances in the Mathematics Institute were very regularly timed. But it was a surprise to some of us, after his untimely death from cancer in the summer of 2006, to find that he had developed a deep interest in poetry and had started to write his own. Given that shared interests in literature, film, theatre, art and music were clearly a factor in the original bond between Bill and Benita, and an important part of life in the Parry household, perhaps it should not have been a surprise. The subjects of the poems combine the natural world, childhood memories and war, and at least two feature mathematics. A fine appreciation of beauty is apparent in 'Alexander's Horned Sphere', and excitement jostles with a finely tuned sense of the possibly absurd in 'Argument', about proving a theorem in a lecture (here reproduced with the permission, gratefully acknowledged, of Bill's executors):

> … wonder will this go through,
> questioning this entanglement
> – yet they nod encouragement.
> Then the final crux; the ropes relax and fall.
>
> His reward: two smile, maybe three,
> and one is visibly moved.
> Q.E.D., the theorem is proved.
>
> This was his sole intent.
> Leaving the symbols on the board,
> he departs with a swagger of achievement.

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The frontispiece photograph was taken in 1968 by Professor Konrad Jacobs at the Mathematisches Forschungsinstitut Oberwohlfach (MFO), and is reproduced courtesy of the Archives of the MFO.

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