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Elected FRS 1957

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Sir Harry Pitt worked (as H. R. Pitt) with Norbert Wiener in 1938 on Wiener's general Tauberian theory. Mathematically, he is best known for Pitt's form of Wiener's Tauberian theorem, and as the author of the first (1958) monograph on Tauberian theory. He is otherwise best known for having been Vice-Chancellor of Reading University from 1964 to 1978.

Introduction

Sir Harry Pitt was a pure mathematician most prominent for his work on Tauberian theorems. A pupil of G. H. Hardy FRS, he was much influenced by Norbert Wiener; his best-known contribution is Pitt's form of Wiener's Tauberian theorem. His first book (30)* of 1958 was until 2004 the only monograph on Tauberian theorems. Pitt was Vice-Chancellor of Reading University from 1964 to 1978, for which he was knighted on his retirement in 1978. He was a kind and considerate man, a devoted husband (for 63 years) and father (of four sons).

Background

Harry was born on 3 June 1914 at Greets Green, West Bromwich, Staffordshire. He was the only son of Harry Pitt and Florence Harriet (*née* Draper). His sister Sybil was born in 1921 and died in 2003. Harry Pitt senior had no opportunity for formal education after the age of 13 years, when he left school. After completing his engineering apprenticeship he went into

* Numbers in this form refer to the bibliography at the end of the text.

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motor-car engineering and was for many years in charge of the inspection and testing department of the Bear Motor Car Company.

H. R. Pitt's ancestors were small farmers and craftsmen.

Education and early life

H. R. Pitt was educated at Greets Green Primary School (1919–21), Church of England School, Wall Heath (1921–24), and King Edward's School Stourbridge (1924–32). He left the latter with a Governor's Exhibition, a Staffordshire County Major and a State Scholarship and a major open scholarship to Peterhouse, Cambridge.

Harry has written (in his Personal Record lodged at the Royal Society):

> King Edward's had a large sixth form and, particularly in the early thirties, a very high academic standard. Teaching in Science and Mathematics was excellent and I do not regret the fact that I spent my last four years at school reading little else. My response to teaching in humane studies was less satisfactory and my early introduction to History, Literature and Languages left me with a positive distaste for these subjects which lasted well into my undergraduate years. I have since tried to remedy this distortion. This may have been due to the fact that I was younger by a year and a half than the average for my form and was too immature to benefit from the sort of approach which was then usual. On the other hand, many of my contemporaries with a scientific bent had similar difficulties, and I have come to the conclusion that the normal school examination approach to general education was (and still is) quite the wrong one. A high degree of specialization is still imposed on pupils in sixth forms, but moves to allow a broader range of study are making progress, albeit slowly. My own experience convinces me that some concentration of study, not necessarily so extreme, is appropriate for some children and, when it is no longer imposed, should not be forbidden.

Pitt was in Peterhouse as an undergraduate from 1932 to 1935, passing with first-class honours in Parts I and II and with distinction in Part III of the Mathematical Tripos and graduating with a first-class BA in 1935.

His tutor and supervisor was J. C. Burkill (FRS 1953), and Harry says that Burkill's attitude and approach made a profound impression on him. He was also strongly influenced by lectures, on functions of a complex variable by A. E. Ingham (FRS 1945), almost periodic functions by A. S. Besicovitch (FRS 1934), theory of functions by J. E. Littlewood FRS and divergent series by G. H. Hardy FRS. It was the latter who determined the direction of Pitt's interest while a research student (1935–36) and a Bye-Fellow (1936–39) at Peterhouse. Hardy's supervision resulted in Pitt's PhD in 1938, for a thesis on Tauberian theorems. These later became the subject of his first book (30). Pitt also gained a Smith's Prize in the same year. He was a Choate Memorial Fellow at Harvard University from 1937 to 1938.

Career

In 1939 Pitt moved to Aberdeen University as an assistant lecturer. In 1942 he moved to London for work at the Air Ministry and the Ministry of Aircraft Production for three years, under H. E. Daniels (FRS 1980). Within the Royal Air Force (RAF) Coastal Command he used probability theory and the newly developing subject of operational research to devise methods of attacking German U-boats and the most efficient use of RAF resources such as fuel. He refused a commission because he felt that it would limit his freedom to influence senior officers.

In 1945, with the war ended, Harry was appointed Professor of Mathematics at Queen's University, Belfast, at the age of 31 years, jumping from the bottom of the academic ladder to the top. Clearly his wartime contribution and several distinguished papers had a key role.

In 1950 Pitt moved to the University of Nottingham as Professor of Pure Mathematics. The Vice-Chancellor was Bertrand Hallward, with ambitions to develop the university. Pitt succeeded H. T. H. Piaggio, whose textbook on differential equations is still useful (Piaggio 1952). Applied mathematics was coming to the fore in this period, and Pitt persuaded the university to establish a chair in this subject, to which Rodney Hill (FRS 1961) was appointed in 1953.

Pitt encouraged the Student Mathematical Society, bringing in such speakers as Mary Cartwright DBE FRS and Charles Coulson FRS.

Mike Sewell and Margaret Joules have written of his inspiration to students, through the clarity of his lectures introducing the ε,δ style of analysis. His enthusiasm and love for the subject permeated his lectures. So when the more senior students set out to produce a mathematical magazine they called it 'Epsilon'. *The Times* devoted 11 column inches to a complimentary review of the first issue. Pitt's door was always open to any student needing help or advice. When Michael Sewell wanted to change from mining engineering to mathematics with inadequate prerequisites in the latter, Pitt made it possible. He also supervised the PhD of Clive (later Sir Clive) Granger, Nobel laureate in Economics in 2003. Another challenge that Pitt had to meet was to see that the new Mathematics and Physics Building, which he had played a big part in planning, was ready for occupancy on the scheduled date.

Nottingham University made full use of his administrative skill. He became in turn a member of Council, Vice-Dean and Dean of the Faculty of Science, and Deputy Vice-Chancellor.

Pitt spent the year 1962/63 on leave of absence as Visiting Professor at Yale University.

Reading

Harry Pitt was Vice-Chancellor of Reading University from 1964 to 1978. It was a period of rapid expansion for universities, in which Reading participated fully. During Pitt's period of office, student numbers grew from 2000 to 6000. Growth has continued ever since, and the numbers now stand at 17 000 and are still rising.

There was an interregnum of 18 months between the summer of 1963, when Wolfenden left to become the Chairman of the University Grants Committee, and Pitt's arrival in 1964. The Acting Vice-Chancellor, Professor J. M. R. Cormack, set up a Committee of Deans to help him, and this committee continued with increased strength under Pitt, dealing with expansion, financial and other policy matters. The Deans were elected by the Faculties, and so Reading University enjoyed a truly democratic government. This was very much Pitt's wish. He wanted to be *primus inter pares*. One of his first acts was to ask Professor Ronald Tuck, Senior Steward of the Common Room, to remove the Vice-Chancellor's chair from the dining room. Pitt's authority was to be without artificial aids. This meant that committee meetings took rather longer than under Wolfenden, but a genuine communal decision was reached in the first instance with senior academic staff. Michael Sewell says that Pitt conducted meetings in such a way that the participants realized only afterwards that the conclusions reached were what Harry Pitt had wanted to happen all along. Perhaps the system resulted in too many departments. Now physics, sociology and most of engineering are closed.

However, during the student troubles of the early 1970s, Pitt, displaying masterly inactivity, weathered the storm well, although at one stage the Vice-Chancellor and the Registrar were locked up by the students and had to escape with a spare key.

A Department of Applied Statistics was started in 1963, with separate funding.

Harry's wife, Catherine Lady Pitt, played a full role in all social staff and student activities. A perhaps apocryphal story has it that Harry was once introduced as the Vice-Chancellor's wife's husband. They generated enormous liking and respect. He was calm and thoughtful as a Vice-Chancellor and as a man.

The university absorbed the National College of Food Technology at Weybridge in the 1960s, and also the College of Estate Management in South Kensington.

During Pitt's period as Vice-Chancellor the move to the large parkland setting at Whiteknights was completed, so there were many opportunities and associated duties for Pitt. He had the skills and diplomacy that were needed.

During the student disturbances many colleagues noted that the registration plate of Pitt's car began with the letters WOE, but the university came out strongly from this period and many new buildings were completed.

Pitt served on many committees, of which the following is a selection. He was Vice-Chairman of the Committee of Vice-Chancellors and Principals (1976–77), President of the Institute of Mathematics and its Applications (1983–85), Chairman of the Universities Central Council for Admissions (1975–78), and a member or chairman of many other bodies such as the governing body of Reading School, chairman of Section A of the Royal Society (1960–61) and of the Royal Society Education Committee (1980–85). Pitt was knighted in 1978, the year in which he retired from the Vice-Chancellorship. He received honorary doctorates from Aberdeen and Nottingham (1970), Reading (1978) and Belfast (1980).

Retirement and family

On Pitt's retirement from being Vice-Chancellor, he and Lady Pitt moved to a house close to the campus, but they took care never to interfere with Harry's successor. They moved to Epsom in 1992 and to Derby in 2002. In his last three years he was wheelchair bound, having had a fall in Epsom and thereafter problems with his balance.

During his long life Harry was always a devoted family man. He had a strong bond with his sister Sybil (1921–2003). He was married on 5 April 1940 to Clemency Catherine Jacoby, second daughter of Henry Charles Edward Jacoby MIEE and Bertha (*née* Dubois). Henry Jacoby was for many years a member of the research staff of the General Electric Company and contributed substantially to the early development of the alternating-current motor. Harry and Catherine stayed together for 63 years, until Catherine died in 2004. They had four sons, Mathew (1945), John (1947), Daniel (1954) and Julian (1958).

Harry has said that his home life was very happy and secure, and so was the life that he and Catherine provided for their sons. In return the sons have clocked up a hundred years of happy marriage between them. They had lovely parties, especially on New Year's Eve, when they acted out little sketches, doing impressions of their colleagues. They went on marvellous family holidays, especially near Snowdonia and Cader Idris, from their cottage in Abergynolwyn. Harry would climb Cader, 2927 feet, well into his late seventies.

Harry had cycled in Czechoslovakia in 1938, and took the family on European holidays after the war. He would always take Catherine's side, stayed calm and never got cross.

Harry had total personal integrity. If he said he would meet the family at the end of a climb with the car and a Thermos flask of tea he would always be there. He was practical as well as a theoretical pure mathematician, making a valve radio in the 1920s. Then in the 1960s he helped Daniel do the same, tactfully suggesting a missing connection to make the set work.

He was never pompous, and it was said of him by one of his colleagues that 'his power was only equalled by his modesty'. He was shrewd in the ways of the world. His advice on restaurants was to choose the cheapest menu at the most expensive place, and in considering whether to purchase a house, to think about what you can change (so don't let that put you off), and what you cannot (so, if it matters to you, stay clear).

Pitt was 'a communicator extraordinaire', able to convey difficult concepts and to summarize complex histories. However, he also listened to the views of others, felt where they were coming from and so, in many walks of life, resolved conflict.

As his sons have said, Harry Pitt was a man blessed with all the 'gifts of the spirit' listed by St Paul in his letter to the Galatians (5:22): 'love, joy, peace, patience, kindness, goodness, faithfulness, gentleness and self-control'.

Mathematical work

Tauberian theorems

Pitt was a research student under G. H. Hardy from 1935 to 1938. It was Hardy who introduced Pitt to the subject of Tauberian theorems, in which he was to do his deepest work.

The precursor of Tauberian theorems is Abel's continuity theorem for power series (1827)—that if a power series $\sum_0^\infty a_n x^n$ converges for $x = 1$, its value converges to $\sum_0^\infty a_n$ as its argument x increases to 1. Writing

$$s_n := \sum_{k=0}^{n} a_k$$

for the partial sums and using partial summation, this may be recast as saying that

$$s_n \to s \qquad (n \to \infty)$$

implies

$$(1-x)\sum_{n=0}^{\infty} s_n x^n \to s \qquad (x \uparrow 1).$$

The converse implication is false, as examples readily show. However, in 1897 Alfred Tauber proved a partial converse, under the additional condition $a_n = o(1/n)$, improved by Littlewood in 1911 to $a_n = O(1/n)$ (Tauber 1897; Littlewood, 1911). Hardy and Littlewood worked on results of this type from 1913 onwards, and introduced the term 'Tauberian theorem' for them. The conditions above are the prototypes for *o*- and *O*-Tauberian conditions respectively; the *O*-case is typically much harder. Hardy and Littlewood studied specific summability methods—those of Cesàro, Abel, Euler, Borel and Riesz, for example—and relatives such as Laplace transforms. The Hardy–Littlewood approach to the Tauberian theorem for Laplace transforms was greatly simplified in Karamata (1930).

The whole area of Tauberian theorems was revolutionized in 1932 by Norbert Wiener's work, which was quite general. Wiener worked with convolutions $\int_{-\infty}^{\infty} k(x-t)f(t)\mathrm{d}t$, where k is a Lebesgue-integrable function, $k \in L_1$, thought of as a kernel, and f is a function—bounded, say (so that the convolution exists). Wiener's favourite tool was the Fourier transform, and as $k \in L_1$ its Fourier transform $\hat{k}(t) := \int e^{ixt} k(x)dx$ exists, at least for t real. The key to Wiener's approach was his approximation theorem, or closure theorem ('Wiener's theorem'): that linear combinations of translates of k are dense in L_1 if and only if $\hat{k}$ has no real zeros (Wiener 1932, ch. I; Wiener 1933, ch. II). From this, one obtains Wiener's general Tauberian theorem—that for such k and f, if

$$\int_{-\infty}^{\infty} k(x-t) f(t) dt \to A \int_{-\infty}^{\infty} k(t) dt \qquad (x \to \infty),$$

then also

$$\int_{-\infty}^{\infty} g(x-t) f(t) dt \to A \int_{-\infty}^{\infty} g(t) dt \qquad (x \to \infty)$$

for any function g in L_1—by an approximation argument. Here the Tauberian condition is $f \in L_\infty$. This approach is very fruitful, with many extensions and variants; almost all known Tauberian theorems for special kernels or summability methods could be deduced from these general results.

Wiener had spent time at Cambridge, both as a student in 1913–14 (Wiener 1953, XIV) and as a visiting professor in 1931–32 (Wiener 1956, ch. 7), and knew both Hardy and Littlewood, and their work, well. Hardy suggested that Pitt should study Wiener's work on Tauberian theorems. In Pitt's final year as a PhD student, 1937/38, he was awarded a Choate Memorial Fellowship at Harvard University. There he worked with D. V. Widder and with Wiener, a professor of mathematics at Massachusetts Institute of Technology, also in Cambridge, Massachusetts. In 1938 Pitt received his Cambridge PhD. It was his *annus mirabilis*, during which he published eight papers, a quarter of his whole corpus.

Generalized harmonic analysis (GHA) (12)

Slightly predating the Wiener Tauberian theory, and leading naturally to it, is Wiener's theory of generalized harmonic analysis (GHA; Wiener 1930). This is devoted to the harmonic analysis of functions that need not be periodic (as in Fourier series), nor in L_2 (as in the Parseval–Plancherel theory), nor in L_1, the case Wiener studied by systematic use of the Lebesgue integral. It is crucial to prediction theory and the spectral analysis of time series (see, for example, Doob (1953), ch. XII, which is based on Wiener's work). Another prime use is for almost periodic functions, the prototypes of functions with discrete spectrum. Under suitable conditions, a function f may be represented as a Fourier–Stieltjes transform $f(x) = \int_{-\infty}^{\infty} e^{-iyx} ds(x)$, where s may be obtained from the function

$$\sigma_y(x) = \frac{1}{2\pi y} \int_{-\infty}^{\infty} f(x-u) \left(\frac{\sin \frac{1}{2} yu}{\frac{1}{2} u} \right)^2 du.$$

General Tauberian theorems (5, 6, 7, 9, 14, 16, 24)

Pitt's first major work here was his long paper (5) on general Tauberian theorems, written in 1937 but published in 1938 (an announcement is given in (6)). One aim was to extend Wiener's methods to some areas of Tauberian theory, such as gap or high-indices theorems (treated below), in which they had not been fully used. But the most important contribution was to show that Wiener's general Tauberian theorem followed from a variant, in which the above hypotheses are retained: there is an additional Tauberian condition of slow decrease,

$$f(u+x) - f(x) \to 0 \qquad (x \to \infty, \quad u \geq 0, \quad u \to 0),$$

and the conclusion becomes

$$f(x) \to A \qquad (x \to \infty).$$

Tauberian conditions of this type were introduced by Robert Schmidt in 1925 (Schmidt 1925), and were used by Wiener to pass from the conclusion of his general Tauberian theorem, a statement on asymptotic behaviour of convolutions, to conclusions on pointwise convergence. Thus Pitt's result is equivalent to Wiener's general Tauberian theorem.

In the first textbook account of Wiener's Tauberian theory, Widder calls Pitt's result *Pitt's form of Wiener's theorem* (Widder 1941, V.10) and uses it to derive Wiener's general Tauberian theorem (Widder 1941, V.11). In Hardy's book on the subject (Hardy 1949, notes to ch. XII), he points out that much of Widder's treatment, and of his own, is based on Pitt's work. Later textbook accounts are in Bingham *et al*. (1987, ch. 4), where Pitt's form of Wiener's theorem is called the Wiener–Pitt theorem, and in Korevaar (2004, ch. II).

Slow decrease, and slow increase, defined similarly, are the prototypes of *one-sided* Tauberian conditions, in the real case (thus in Littlewood's theorem above, it is enough to have $a_n = O_L(1/n)$, or na_n bounded below). This is typically the case with real non-negative kernels, as Pitt showed in (7). In (14), the sequel to (5), Pitt develops one-sided Tauberian conditions further, using his work (12) on GHA.

Much of (5) is devoted to *Tauberian classes*, a classification of Tauberian conditions. The ideas introduced there were so clearly important that they came under detailed scrutiny, and it turned out that the paper contained errors. These were pointed out by the American analyst R. P. Agnew; Pitt wrote his brief paper (16) to correct his results.

In addition to Fourier transforms, the Fourier–Stieltjes transform (FST),

$$\hat{K}(t) := \int_{-\infty}^{\infty} e^{ixt} dK(x),$$

is also important in the Wiener Tauberian theory, for functions K of finite variation (all integrals here are Lebesgue–Stieltjes, and so absolutely convergent). Here the basic role of non-vanishing of the Fourier transform (on the real line) is replaced by boundedness away from zero. Note that the presence of a discrete or a continuous singular component in the Lebesgue decomposition of K is necessary here: were K absolutely continuous, its transform would tend to zero at infinity by the Riemann–Lebesgue lemma, so could not be bounded away from zero. It turns out that a discrete component is essential, and that if a continuous singular component is present, it must be dominated by the discrete component, in the sense of the following condition. If $D(t) := \sum_n d_n e^{it\lambda_n}$ is the Fourier transform of the discrete component, dq is the continuous singular component if present, then one requires

$$|q| := \int |dq(x)| < \inf\{|\hat{K}(t)| : t \in \mathfrak{R}\},$$

the *Wiener–Pitt condition*. In Pitt's first paper with Wiener, (9), it is shown that if this condition holds, and also $\inf\{|\hat{K}(t)| : t \in \mathfrak{R}\} > 0$, then the reciprocal $1/\hat{K}(t)$ is itself a FST. Conversely, the Wiener–Pitt condition is necessary here ((9); see also Pitt's book (30), Chapter V, Theorems 6 and 8). It was shown further by Kahane & Rudin (1958) that if $F(.,.)$ operates on FSTs, in the sense that $F(\mathfrak{R}\hat{K}, \mathfrak{I}\hat{K})$ is a FST whenever $\hat{K}$ is, then F is real entire. Thus the example $1/z$ of the reciprocal above is excluded.

The existence of such measures, whose FSTs are bounded away from zero but are nonetheless not invertible, is called the *Wiener–Pitt phenomenon*. It has attracted much interest in modern harmonic analysis, and its natural setting is now known to be non-discrete locally compact groups, rather than the line as in (9). Textbook accounts are in Rudin (1962, Th. 5.3.4) and Benedetto (1975, Th. 2.4.4) (see also Hewitt & Ross 1970, pp. 519 and 574). The algebraic aspects were first studied by Šreider (1950), who also corrected the Wiener–Pitt proof in (9) (Kakutani 1986, pp. 116 and 400; as remarked there, the relevant ideal structure is extremely complicated). Accordingly, some authors call this the Wiener–Pitt–Šreider phenomenon. Quantitative versions are now known, and there are interesting links with the corona theorem in Hardy space theory (see Nikolski 1999). We return to algebraic aspects and ideal structure later in connection with Pitt's book (30).

In (24), Pitt considers transforms $g(u) = \int k(u, y)s(y)\mathrm{d}y$ not necessarily of convolution type. Results along the lines of (5) are given.

Mercerian theorems (8, 17)

It was shown in Mercer (1907) that for a sequence s_n,

$$\frac{1}{2}s_n + \frac{1}{2}\cdot\frac{1}{n}\sum_{k=1}^{n} s_k \to s \quad (n \to \infty) \quad \Leftrightarrow \quad s_n \to s.$$

The point of interest here is that, although the statement has some of the features of a Tauberian theorem, no Tauberian condition is needed. The result is thus not Tauberian. Hardy and Littlewood introduced the term *Mercerian* for results of this type, which go from a hypothesis on a function (or sequence) and some average or smoothed form of it to a conclusion on the function alone, with no Tauberian condition. The Wiener Tauberian theory was applied to Mercerian theorems by Paley & Wiener (1934, IV.18). In (8), Pitt applied his work (9) with Wiener to a study of Mercerian theorems in full generality. This was continued in (17), which also used GHA as in (12). This work later formed the basis of chapter V of Pitt's first book, (30).

Ikehara's theorem (13)

With $\pi(x)$ the function that counts the number of primes p up to some positive value x, the prime number theorem (PNT) is the statement that $\pi(x) \sim x/\log x$ as $x \to \infty$. This was conjectured by Gauss, and only proved in 1896—independently by Hadamard and de la Vallée Poussin, both using complex analysis (Hadamard 1896; de la Vallée Poussin 1896). Their work, and that of Landau, showed the crucial role played here by the Riemann zeta function $\zeta(s) := \sum_{n=1}^{\infty} n^{-s}$. Two properties are particularly relevant here: that ζ has a simple pole at $s = 1$ of residue 1, and that ζ has no zeros on the line $\Re s = 1$. Chapter III of Wiener (1933) gives an account of PNT and includes two proofs via the Wiener Tauberian theory. The first uses Lambert series, as in Landau's treatment. The second uses Ikehara's theorem. S. Ikehara was Wiener's first PhD student; his paper of 1931 proves PNT via Wiener Tauberian theory, essentially by subtracting off the pole of the zeta function at $s = 1$ and analysing the remainder (Ikehara 1931). In Pitt's second paper with Wiener (13), Ikehara's theorem is generalized.

For recent developments, using the language of Schwartz distributions and applied to the twin primes conjecture, see Korevaar (2005).

Abel and Cesàro methods (27)

Pitt was also interested, as were Hardy and Littlewood before him, in special summability methods. Prominent among these are the Abel method on power series, and the family of Cesàro methods based on arithmetic means. The Tauberian theory for all these methods is very similar. However, in (27), Pitt finds a condition that is Tauberian for the Cesàro method but not for the Abel method.

The Borel method (5, 28)

One says that a sequence s_n converges to s in the sense of the *Borel summability method*, $s_n \to s\ (B)$, if $\sum_{n=0}^{\infty} s_n.e^{-x}x^n / n! \to s$ as $x \to \infty$. The Borel method is perhaps the most important summability method after those of Cesàro and Abel. However, whereas the Wiener Tauberian theory concerns convolutions, and both the Cesàro and Abel methods are reducible to convolutions (for the latter, the x^n becomes e^{-xn} after a change of variable, and the product xn becomes the argument $x - t$ for a convolution after a further change of variables), the Borel method does not lend itself to reduction to convolution form so readily. One needs an approximation procedure to effect this reduction; see §4.3 of (30) for a textbook account, or (5) for the original paper. Pitt's work involves the Borel method in two additional ways. The first concerns the classical 'Borel–Tauber theorem'—that $s_n \to s\ (B)$ and $a_n = O(1/\sqrt{n})$ or $O_L(1/\sqrt{n})$ imply $s_n \to s$. In (28), Pitt proves the closure in $L_1(\mathfrak{R})$ of translates of e^{-x^2} by elementary means; that is, without use of the Wiener closure theorem (the Fourier transform of e^{-x^2} is an exponential, so non-zero, so with the Wiener closure theorem there is nothing to prove). The second concerns the 'Borel gap theorem'. A gap theorem, or high-indices theorem, is a result in which, if the terms a_n of a series are known to vanish except on some subsequence n_k, convergence of the sequence $s_n := \sum_0^n a_k$ under some summability method implies ordinary convergence. Here the point is that there is no Tauberian condition (beyond a_n vanishing off the sequence $n = n_k$). Sometimes such additional Tauberian conditions are imposed; such results may still be called gap theorems, but results of the earlier type are then called *pure gap theorems*. A textbook account of gap theorems from the point of view of the Wiener theory was given by Wiener's pupil Norman Levinson (Levinson 1940). A pure gap theorem for the Abel method was obtained by Hardy and Littlewood in 1926, the gap condition being $n_{k+1} - n_k \geq hn_k$ for some positive h (Hardy & Littlewood 1926). Examination of the Borel method suggested that the relevant gap condition here was $n_{k+1} - n_k \geq h\sqrt{n_k}$ for some positive h. One of the aims of Pitt's first major paper (5) in this area was to bring the field of gap theorems more into line with the Wiener Tauberian theory. Unfortunately, (5) contained further errors, beyond those addressed in (16). Not detecting the error relevant to the Borel gap case, Meyer-König (1953) gave a 'Borel gap theorem' under the additional Tauberian condition $s_n = O(e^{cn})$ for some positive c. The proof, once Pitt's error was detected, was incomplete. The result was proved under the stronger additional Tauberian condition $s_n = O(e^{c\sqrt{n}})$ for some positive c; see (30), Th. 31. However, even this reveals less than the full truth. The pure 'Borel gap theorem' is actually true—that is, *no* additional Tauberian condition is needed. But this was proved only considerably later, by Gaier (1965). This followed several earlier partial results, for example by Erdős for the related but easier case of the Euler method. Other approaches were later given by Gaier himself, by Mel'nik and by Turán. The matter is subtle. For example, if one changes the continuous variable $x \to \infty$ in the definition of the Borel method to an integer $n \to \infty$ (the 'discrete Borel method'), no pure gap theorem holds, as was shown by Meyer-König and Zeller by functional-analytic methods.

Elementary proof of the prime number theorem (29)

One striking feature of the approaches above to PNT is that they involve complex analysis, because to formulate the problem one needs only integers, and to formulate its solution one needs only reals (via the logarithm). It had been a standing problem of great interest to prove PNT by 'elementary' methods—that is, avoiding complex analysis (it was clear that any such proof would be harder than the existing ones, so the word 'elementary' must be used with some care here). A very ingenious elementary proof of PNT was found in 1949 by Erdős and by Selberg (extended to the PNT for primes in arithmetic progression by Selberg) (Erdős 1949; Selberg 1949*a*, *b*). In his last paper on Tauberian theory (29) in 1958, Pitt obtains an elementary proof of PNT, using the Stieltjes kernel $K(x) := \sum_{\log p \leq x} \log p / p$, essentially along the lines followed by Selberg. This work was done while Pitt was writing his first book ((30), see below), which also appeared in 1958. The approach of (29) is also used in the last section of (30).

Analysis

Inequalities (1, 3, 10, 11)

It was natural for Pitt to become interested in inequalities, in view of the book by Hardy *et al.* (1934), and two of his earliest papers are in this area. In (1), he considers infinite double sums $\sum_{i,j} a_{ij} x_i y_j$, obtaining results on the space $[p,q]$ $(p, q > 0)$ of sequences with $|x|_p \leq 1$, $|y|_q \leq 1$. In (3), he obtains integral analogues of results of Hardy and Littlewood on convolutions $c = (c_n)$ of non-negative sequences $a = (a_n)$, $b = (b_n)$, of the form $C \leq KAB$, where $A = \sum_1^\infty n^{-1}(n^\alpha a_n)^p$ and B, C are defined similarly in terms of further parameters q, r, β and γ. In (11) he links $a = (a_n) \in l_p$, for $p \in [1,2]$ with $f \in B_q$, the space of Besicovitch almost periodic functions with index q, where p, q are conjugate and f has Fourier series $\sum a_n e^{i\lambda_n x}$ with λ_n real.

In (10), Halperin and Pitt, following work by Halperin (Halperin 1937), consider the subclass D_0 of $L_p(a,b)$ consisting of functions f satisfying differential recurrence relations of the form $f_0 = q_0 f$, $f_{r+1} = f_r' + q_{r+1} f$ $(r = 0, 1, \ldots, n-1)$, with the functions q_k suitably restricted, and $f_r(a) = f_r(b) = 0$. This space is shown to be dense in $L_p(a,b)$. Operators T of the form $Tf = \sum_0^n p_r f_r + cf$, and their adjoints T^*, are studied, via inequalities on L_p and L_q. Pitt's collaborator here was the Canadian mathematician Israel Halperin (1911–2007), von Neumann's only graduate student and one of the founders of the study of von Neumann algebras.

Fourier analysis (2, 4, 23)

Let f be a function with Fourier coefficients c_n. For $p \in (1,2]$, Paley obtained comparisons between $|f|_p$ and $(\sum_n |c_n|^p n^{p-2})^{1/p}$ (Zygmund 1959, §XII.5), following earlier work by Hardy and Littlewood (Hardy 1969, comments, pp. 313 and 399). In (2), Pitt extended these results, and gave analogues for power series. 'Pitt's inequality' has recently been sharpened by Beckner (1995, 2008).

In (4), Pitt gives a simplified proof of a result of Cameron (1937), that if f is almost periodic with an absolutely convergent Fourier series and g is complex analytic on the range of f, then $g(f)$ also has an absolutely convergent Fourier series.

Three classes of interest are the classes of functions that are (i) Fourier transforms of integrable functions, (ii) Fourier–Stieltjes transforms of functions of finite variation, and (iii) periodic functions with absolutely convergent Fourier series. Consider the three classes of functions equal in some neighbourhood of a point, x_o say, to a function in each of the classes (i)–(iii). It is shown in (23) that these three classes coincide—that is, that the local aspects of the three properties are the same.

Integro-differential equations (20, 22)

The equations studied are of the form

$$\sum_{r=0}^{R} \int_{-\infty}^{\infty} f^{(r)}(x-y) \mathrm{d}k_r(y) = g(x)$$

in (22), with the corresponding homogeneous equation in (20), the summands on the left being convolutions of the derivatives of a function f with Stieltjes kernels k_r. Crucial here is the function $K(\omega) = \sum_0^R \omega^r \int e^{-\omega y} dk_r(y)$. Under weak conditions, solutions f have expansions of the form $\sum A_n(x)e^{\omega_n x}$, with A_n polynomials and ω_n the zeros of K.

Conditions for the convergence almost everywhere of Fourier series

Pitt mentions in his autobiographical notes to the Royal Society that 'The last of these problems occupied the major part of my time for several years. My efforts so far have been completely unsuccessful. [The problem was solved by Carleson in 1964.]' For mathematical background to the Carleson–Hunt theorem, see Carleson (1966) and Hunt (1968). This episode, clearly coming between his election as FRS in 1957 and his becoming Vice-Chancellor of Reading in 1964, may partly account for Pitt's motivation in his career change.

Probability, statistics, ergodic theory

Probability (15, 19, 21, 26)

Wiener was a great probabilist as well as a great analyst—his Wiener measure of 1923 is the key to a mathematical treatment of Brownian motion, for example. Through Wiener, Pitt became interested in probability, which achieved its modern measure-theoretic form through the work of Kolmogorov (1933). Pitt's first paper in this area, (15), concerns stochastic processes with stationary independent increments, or Lévy processes. He focuses on the fairly simple case, of compound Poisson processes (only finitely many jumps in finite time intervals). This is extended from the real-valued to the G-valued case in (19), where G is a locally compact abelian group. In (21), on storage models, Pitt considers a stochastic model for the amount held in store or inventory, giving a comparison between two different replacement policies. As the paper dates from 1946, this was presumably motivated by Pitt's wartime work on operations research. In (26), Pitt addresses the question of defining measures in function space. The motivation is the theory of stochastic processes, where (as with Brownian motion, or Wiener measure) the set of time-points is uncountable. Care needs then to be taken to ensure that sets of interest are measurable ('are events'—that is, that their probabilities are defined). Foundational work on such problems was done by J. L. Doob in a series of papers, the earliest of which were known to Pitt, but the area received a definitive treatment only in the classic book Doob (1953). As S. Kakutani points out in his review (Kakutani 1953), (26) contains some errors.

Statistics (25)

Pitt's paper (25) addresses foundational questions in statistical decision theory. He starts from work of Abraham Wald in 1939, citing also work of the 1920s by Fisher on maximum-likelihood estimation and by Neyman and Pearson on testing statistical hypotheses (Wald 1939). Unfortunately, later work by Wald is also relevant, as was pointed out by J. Wolfowitz in his review (Wolfowitz 1949). The game-theoretic ideas of von Neumann & Morgenstern (1944) had a decisive influence on the area, as was seen in the books by Wald (1950) and Blackwell & Girshick (1954), and later by many others.

Pitt's interest in statistics had important consequences in econometrics, through his supervision of the doctoral thesis of C. W. J. (later Sir Clive) Granger at Nottingham (PhD 1959; Granger was Pitt's only doctoral student, according to the Mathematics Genealogy Project). Granger and R. F. Engle received the Nobel Prize in Economics in 2003, for their work on cointegration and other aspects of econometric time series analysis. For background, see Engle & Granger (1987) and the books by Granger & Hatanaka (1964) and Granger & Newbold (1986).

Pitt's interest in statistics had effects in his later career as Vice-Chancellor of Reading University. The first head of the Department of Applied Statistics there, Professor Robert Curnow, writes (Curnow 2006, p. 12):

> The title of the Department needs explanation. The original title was to be Statistics. Shortly after his arrival the new Vice-Chancellor, Harry Pitt, a distinguished mathematician and probabilist, told me that statistics was a branch of mathematics, as in many senses it is, and we should therefore be a part of the Department of Mathematics not an independent Department. Fortunately we had many friends in the University who appreciated our attempts to teach courses on statistical methods appropriate to their students. They feared that this and the consultancy service would be lost if we were administratively part of a mathematics department. These friends convinced a reluctant Vice-Chancellor that we needed independence. He insisted on the compromise that the title of the Department should be Applied Statistics. A year or so later Harry Pitt told me that he now realized that we were mathematicians by training and our interests and teaching covered the underlying mathematics of our subject as well as its applications. We could therefore drop 'applied' from our title. He was surprised but, I think, content when I told him that we now liked the title of Applied Statistics….

The interface between mathematics and statistics is interpreted in different ways in different universities in the UK; all three of the obvious solutions—separate departments, statistics within mathematics as a formal entity, and statistics treated as part of mathematics on a par with pure or applied—are found.

Ergodic theory (18)

Wiener's interests were remarkably broad, and included ergodic theory, to which he made several contributions (see, for example, Krengel 1985), and through him Pitt became interested in the area. In (18), he gave what Dunford (1943) describes as a new and elegant proof of the Yosida–Kakutani maximal ergodic theorem. From this he derives the Birkhoff–Khintchine almost-everywhere ergodic theorem, without use of the mean ergodic theorem, and Wiener's ergodic theorem in n dimensions (Krengel 1985, p. 203). He also obtains the first random ergodic theorem. For later work by Ulam and von Neumann and by Kakutani, see Kakutani (1986, pp. 364–378 and 445–446); for textbook accounts, see Halmos (1956) and Krengel (1985, §8.2.3).

Books, etc.

Tauberian theorems (30)

This classic book is the first monograph exposition of Tauberian theorems. As the author said, by 1958 the field had grown to the extent that one could not cover it fully in a book of this length (174 pages). Accordingly, Pitt limits himself to 'the topics and methods which follow most naturally from the work of Hardy, Littlewood and Wiener; and my debt to them will be apparent.' After a brief introduction, chapter II deals with elementary Tauberian theorems, chapter III with classical Tauberian theorems (special summability methods, such as Cesàro, Abel and Borel) and chapter IV with Wiener theory. Chapter V is on Mercerian theorems, regarded as a limiting case of Tauberian theorems (this remained the only textbook chapter on the subject until chapter 5 of Bingham *et al*. (1987)). Chapter VI is on Tauberian theorems and the prime number theorem, as in (29).

The Lebesgue space L_1 has the structure of a Banach algebra under convolution, and on taking Fourier transforms, one obtains a Banach algebra under multiplication, in L_∞. Wiener's theorem shows that the presence or absence of zeros in the Fourier transform is crucial, and the presence of zeros is preserved under multiplication (or of zeros in the transform, under convolution). The ideal structure of the Banach algebra is thus relevant. One of the first spectacular triumphs of modern functional analysis, as distinct from classical analysis, was the exploitation of ideal structure in Banach algebras to simplify and extend Wiener Tauberian theory. This was carried out by I. M. Gel'fand in the 1940s, and also by R. Godement. For an early textbook account, see Loomis (1953, p. 85: 'As a corollary of this theorem we can deduce the Wiener Tauberian theorem, but in a disguise which the reader may find perfect'). Pitt refers to Gel'fand (and to Šreider, though not to Godement), but confines himself to the classical methods with which he had himself worked.

Integration, measure and probability (31)

This brief book (110 pages) deals with the three topics in the title, starting from scratch. Unlike (30), which is unambiguously a research monograph, the book reads like a student text. Part one is on integration and measure. Chapter 1, on sets and set functions, is presented in modern notation and terminology (in contrast to (15)). In chapter 2, integration is developed first, and measure is deduced from it. This is the route followed by P. J. Daniell in 1917–20, and later by Bourbaki, but it is not the usual route, and Pitt does not discuss his reasons for choosing it. Chapter 3 includes Stieltjes integrals and convolutions. Part two is on probability. Chapter 4 gives a measure-theoretic framework going as far as conditioning. Chapter 5 covers convergence of random series, infinite divisibility and self-decomposability, and the Poisson process.

Integration for use (32)

This book, also brief (143 pages), is again a student text. In the first three chapters, Pitt covers the basics of integration, including the Lebesgue theorems, Fubini's theorem and the Radon–Nikodým theorem. He again deals with integration first and measure second. Chapter 4 is on geometric theory, including the theorems of Gauss, Green and Stokes. Chapter 5 is on harmonic analysis, and covers Fourier series and transforms, Fourier–Stieltjes transforms and spectra. The final chapter gives a selection of topics on probability.

Obituary: John Charles Burkill (33)

J. C. Burkill (1900–93; FRS 1953) was Pitt's tutor and supervisor at Peterhouse. In addition to his obvious debt to Hardy, Pitt acknowledged a deep debt to Burkill; he was also influenced while at Cambridge by Littlewood, Ingham and Besicovitch. His obituary of Burkill describes Burkill's life and work, and discusses his 27 papers and 6 books under six main headings: integration and differentiation; functions of integrals and the Burkill integral; derivatives of interval functions; the expression of area as an integral; approximate differentiation and extension of the Perron integral; and other topics.

Conclusion

Pitt's output was unusual in that so much of it was done early, in or around the year 1938, when the influence of his early mentors, Hardy and Wiener, was still strong. His name is best known for Pitt's form of Wiener's theorem (5). Pitt's other most important papers include (2) (Pitt's inequality), (9) (the Wiener–Pitt(–Šreider) phenomenon) and (18) (random ergodic theorem). His book (30) served from its appearance in 1958 to that of Korevaar (2004) as the only general monographic treatment of the very important subject of Tauberian theorems. Pitt will also be remembered as the last grandmaster of analysis from the Hardy–Littlewood school (Titchmarsh, Ingham, Offord, Burkill and Cartwright having predeceased him). He will be remembered in the university world for his Vice-Chancellorship of Reading. His colleagues recall a kind man who led by consensus.

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