LESLIE SYDNEY DENNIS MORLEY FRENG FRAes

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Leslie Morley’s research focused on modelling structural behaviour, with particular emphasis on plates and shells. He developed the Morley shell equation, which has been acknowledged as the simplest equation consistent with first-order shell theory. As the finite element method rose to prominence he developed elements for both plates and shells. He then worked on developing a set of new finite elements able to handle complex shell behaviour in both the linear and nonlinear regimes. He also observed that it was possible to augment the finite element solution by using singular solutions to calculate the stress intensity factor at a crack tip in a thin-walled metal structure and thereby to compute crack propagation rates. In undertaking his research Morley probed into the mathematical and physical depths of the problems he confronted, and produced some outstanding and significant results.

EARLY YEARS

Leslie Morley, or ‘Les’ as he liked to be known, was born on 23 May 1924 in Brighton, Sussex. He was the only child of Sydney Victor Morley and Doris May Huntley Morley.

In 1926, when the family moved to Portsmouth, Les’s father was an Able Seaman in the Royal Navy and the family means were modest. By 1937 his father had reached the rank of Chief Petty Officer on the battle cruiser HMS Hood, flagship of the Royal Navy. He specialized in the Hood’s main armament, its 15-inch gunnery, and was highly regarded as one of HMS Hood’s longest-serving ratings. Les recalled that his father was also a leading exponent of the prewar Royal Tournament naval gun competitions, both as a participant and later as an instructor for the Portsmouth team.

During his early childhood Les suffered a serious speech impediment that persisted throughout his life, although it became less evident in later life. This limited his ability to
undertake public speaking or lecturing duties. However, he derived much encouragement in his educational pursuits from his father and mother, who had themselves received only an elementary education. He recalled that as a child he had a fascination for numbers and each evening would keenly prepare his own arithmetical exercises for completion.

His father had developed a passion for trigonometry as part of his naval training in gunnery. His direct influence on Les’s developing mathematical interests was, however, limited by the necessities of his naval service. This required his father’s serving commissions of two and a half years at sea followed by a similar period at home, based in Portsmouth.

Four years after his promotion to Chief Petty Officer, Les’s father was lost in action in the fateful encounter between HMS *Hood* and the battleship *Bismarck* in the Denmark Strait on 24 May 1941, a day after Les’s 17th birthday. For the duration of the war Les elected to remain with his mother at Portsmouth until her untimely death in 1945.

**EDUCATION AND EMPLOYMENT**

Les was 15 years of age at the outbreak of World War II, and Portsmouth, with its Royal Navy dockyard, was regarded by Germany as an important target. At the time he was a student at Portsmouth Northern Secondary School and was offered the opportunity to evacuate with the school to the safer haven of Peter Symonds School in Winchester. It was decided, however, that he should leave school to remain in Portsmouth with his mother, who would have otherwise been on her own while his father was away on active service.

In 1940 Les obtained employment with the Portsmouth-based Airspeed Limited aircraft company and commenced a five-year indentured apprenticeship as a tool room fitter. He started in the machine shop with deburring operations before progressing to work on the capstan lathes.

The next two years of his apprenticeship from 1941 to 1943 were spent in the tool room itself, the centre of precision engineering. Here Les learned the skills and craftsmanship that served him well throughout his life. He gained experience in jig assembly and tool making. He soon developed an interest in the design and development of multiple-action press tools to blank and bend light alloy sheet into intricate components in a single operation.

This period coincided with the commencement of German air raids on Portsmouth, during which Les served with the Home Guard, helping to man the anti-aircraft batteries of rocket guns on Southsea front adjacent to the Dockyard. He retained vivid memories of Messerschmitt fighters coming in over the Isle of Wight to shoot down the defensive balloons. These fighters were soon followed by Stuka bombers taking up formation to dive-bomb the Royal Navy dockyards and the RAF fighter base at Thorney Island. Les recalled that during these raids they would launch some 50 rockets at a time in formation. Each rocket was about 4 feet long and some 3 inches in diameter. He recalled that it was a frightening experience as the rockets left the rails and that the noise of the launch was horrendous! Les always wondered how the German pilots must have felt when these exploded simultaneously in a box-like array in the sky.

His apprenticeship continued with a move, in 1943, to the Airspeed Aircraft Drawing Office to help prepare drawings for the Horsa gliders, which played such a prominent part in the Battle of Arnhem. He then transferred to Airspeed’s Stress Office, which subsequently changed location in 1944 to Christchurch (near Bournemouth) to work on the design and
development of the postwar civilian Airspeed Ambassador airliner. During 1944–45 Les worked on door cut-outs in the fuselage of the Ambassador airliner.

During the course of his apprenticeship Les attended part-time classes at Portsmouth Municipal Technical School. By 1943 he had completed his studies and was awarded the Ordinary National Certificate in Mechanical Engineering, with special reference to Aeronautics. This was followed by part-time studies at Southampton University College, where in 1945 he gained a Higher National Certificate in mathematics, strength of materials and structures, and applied aerodynamics.

By 1945 Les had completed his apprenticeship and had by then, unfortunately, lost both of his parents. His Certificate of Industry from Airspeed Limited dated 23 May 1945 read:

An outstanding Apprentice in every respect. Consistent in every regard and full of initiative. At part time studies he has worked extremely hard under difficult war conditions and obtained the Ordinary National Certificate and the Higher National Certificate during his Apprenticeship. Deserving of every encouragement.

Les continued his employment with Airspeed Limited as a ‘stressman’ in the Stress Office until 1946, when, by chance, he noticed a column in the Daily Mirror that announced the setting up of a new College of Aeronautics at Cranfield. He applied and had the good fortune to gain a place with the first batch of students as well as obtaining a Hampshire County Council grant to pay the fees and maintenance.

From 1946 to 1948 Les was a founder student on the Post Graduate Diploma Course in the Theory of Structures at Cranfield, studying aerodynamics, aircraft design and mathematics, and completed a project entitled ‘Stress analysis of openings in reinforced thin walled cylinders’. He remembered Cranfield with great affection and that Professor W. S. Hemp had provided much inspiration and encouragement both at this time and also in his later research work.

From 1948 to 1949 Les gained employment with the National Luchtvaartlaboratorium (NLR) in Amsterdam as a Research Officer. His work concerned structures research investigating the behaviour of thin-walled plates and shells. He also undertook work on the flexibility of aircraft during landing impact. It was here that Les published his first three original research papers in the Reports Transactions of the NLR.

In 1949, Les returned to the UK and was employed as a Technical Assistant at the Bristol Aeroplane Company, Filton, Bristol. Here he was involved in stress work on the Brabazon Mk 2 fuselage with special reference to the strength of undercarriage frames subjected to concentrated impact loads. He was also involved with determining the flutter speed of highly swept-back wing designs of single-mission expendable aircraft. He joined the staff of the Royal Aircraft Establishment (RAE) at Farnborough as a Senior Scientific Officer in 1952 and remained until his retirement in 1984 as a Deputy Chief Scientific Officer (Individual Merit).

Shortly before he was due to retire from the RAE Les wrote to John Whiteman, Director of the Brunel Institute of Computational Mathematics (BICOM), at Brunel University asking if a position could be found for him; BICOM research concentrates on finite element methods and their applications, including fracture. John obtained contracts for work on shell finite elements and in 1985 was able to appoint Les as a Professorial Research Fellow in BICOM. Professor R. E. D. Bishop FRS, then Vice Chancellor of Brunel University, delighted in saying to John, ‘How on earth did you persuade Morley to join BICOM!’ Les remained at BICOM for 10 years, supervising research students Michael Mould and Tim Bangemann, providing
wise advice and input to the research programme, and being involved in the organizing of the triennial ‘The Mathematics of Finite Elements and Applications’ (MAFELAP) conferences. He related very positively to and had a strong empathy with his students, frequently saying that his time at BICOM was one of the happiest periods of his working life.

This was followed from 1999 to 2002 by a collaborative research venture with Imperial College and the Ministry of Defence. In both posts he continued his research and supervised a succession of PhD students, which he found a particularly rewarding activity.

**RESEARCH ACHIEVEMENTS**

Morley started his research career at the NLR in Amsterdam, where he produced several internal publications on the behaviour of stiffened plates and reinforced monocoque structures. Although this did not result in any external publications, it introduced him to the world of research.

Shortly after Morley joined the RAE in 1952, the UK’s first civil jet airliner, the de Havilland Comet, crashed and became the subject of investigation at the RAE under Arnold (later Sir Arnold) Hall (FRS 1953), the then Director of the Establishment. Initially, the Head of Structures Department, P. D. Walker, thought that the cause might be high stress levels induced by stress waves (or, as Les once expressed it to me, ‘he had a bee in his bonnet’). Morley was asked to look at this possibility and studied the stress waves that occur in a reinforced shell structure disintegrating under internal air pressure as the result of fuselage failure. This was a potentially significant factor in establishing the root cause of the Comet disaster because there was a possibility that stress waves, generated early during the disintegration, might have caused a secondary fracture capable of misleading the accident investigators. Unfortunately, this work was not published, with the exception of one paper on stress waves in a naturally curved rod (3)*, which appeared long after the cause of the Comet crash was fully understood and identified as fatigue failure of the fuselage structure.

After this early diversion, Morley focused his research objectives on the behaviour of plates and shells. This was before the introduction of digital computers, so the limitations in calculating power meant that his early papers on plates and shells were often devoted to the application of methods that were specific to a class of problem. In many of the papers, however, Morley was clearly looking for approaches that had a more general domain of application. One outstanding example is his 1956 paper entitled ‘The approximate solution of plate problems’ (1). Here Morley employed a superposition of particular functions, each satisfying the governing differential equations on the interior of the plate but not on the plate boundary. The lack of compatibility was accounted for by minimizing a potential energy function. This anticipated the approach used in creating finite elements through the application of a variational principle. So it is no surprise that once the finite element method appeared he immediately recognized that this provided the key for obtaining solutions for general analysis problems that arise in the design of structures operating in complex loading environments. He returned to this topic in a subsequent paper (6) in which he derived variational principles for plate bending problems in which the boundary was part clamped or simply supported, or where boundary tractions were prescribed.

* Numbers in this form refer to the bibliography at the end of the text.
Turning his attention to thin shells, Morley showed that the equations that govern the small deflection behaviour of a cylindrical shell could be expressed by a relatively simple equation that improved on that presented by Donnell (2). This equation, known as the Morley shell equation, is acknowledged as the simplest possible equation consistent with the errors of first-approximation shell theory. He then employed the equation to solve the problem of elastic thin-walled cylindrical shells subjected to radial point loads.

Morley’s continued researches in this early period culminated in the publication of his monograph *Skew plates and structures* (5), which has had wide application in aeronautical and civil engineering structures. Although published in 1963, the monograph remains a classic and is still frequently cited. Results from the monograph and those from an earlier paper on the bending of rhombic plates (4) have been extensively used by the finite element community as a source for benchmark cases.

Although his early work was significant, particularly that presented in the monograph, Morley’s reputation was built on his work in developing the finite elements for the solution of thin-walled plates and shells. In the period 1962–65 several papers had appeared that introduced triangular finite elements for plate bending problems. These displacement-based elements could be either conforming (preserving displacement continuity across element junctions) or non-conforming (where such continuity is not preserved); conforming elements tended to exhibit over-stiffness, whereas non-conforming elements did not guarantee convergence. These problems were circumvented by the introduction of equilibrium elements in which the element internal variables were bending moments and inter-element continuity was enforced by Lagrange multipliers modelled by edge displacement, a technique originally introduced by Fraeijs de Veubeke (1965) and exploited by Herrman (1967) and particularly by Allman (1970). Such elements are somewhat complicated and in the case of Fraeijs de Vuebeke and Allman the use of mid-side connection quantities created difficulties in that it gave rise to a rank deficient matrix. Morley sought for simplicity. In satisfying this quest he proposed a non-conforming displacement triangular finite element that was derived from quadratically varying displacements and was therefore a constant-bending moment element (7, 10). Morley showed that this simple element gave identical results to those given by Allman’s element that and this ensured convergence as successively finer meshes were employed. The element also satisfied the patch tests used to ensure the quality of the element’s performance. Morley’s triangular plate bending element proved to be a seminal contribution in the finite element analysis of flat plates and has been employed in numerous finite element packages. It was a key development, which he exploited in working on the creation of thin-shell finite elements. Morley’s flat-facet triangular bending element, which continues to be known in the literature as the ‘Morley element’, has been and continues to be exploited by workers in computational mechanics and particularly in thin-shell behaviour (see, for example, Ming & Xu 2006).

Morley subsequently generalized this element to cover intrinsic geometrically nonlinear situations (20). This generalization represented a considerable achievement because, for the first time, it allowed the calculation of the strain energy employing Fréchet directional derivatives, essential for the rational analysis of the discrete finite element equations governing the stability of equilibrium at singular points in the equilibrium path where, for example, the global tangent stiffness matrix is singular.

Morley recognized that the approach he employed in solutions of plate and shell problems could be adapted to problems with inherent stress singularities. In addressing this problem...
(8) he proposed a modification to the Rayleigh–Ritz method that employed a superposition of analytic solutions and piecewise continuous finite element trial functions. In doing so the finite element solution itself was partitioned into recognizable parts. The implementation resolved into four successive stages:

(i) a first part of the solution is the conventional finite element approximation of the base problem using piecewise polynomials;
(ii) each individual discontinuity or singularity is isolated and its analytical structure described in terms of an analytic solution that satisfies exactly the governing equations over the domain, but not the boundary conditions;
(iii) the boundary conditions and body forces arising from stage (ii) provide further individual problems, each contributing to the overall finite element solution;
(iv) the differences between the analytic functions and their finite element solutions generated in stages (iii) and (ii) provides the extension to the field of definition of the coordinate functions. Their amplitude is determined by minimizing the potential energy to reveal the stress concentration factor.

Morley then employed this approach to evaluate the stress concentration factor for a simply supported square flat plate under a uniformly distributed load with a square hole (9). He then moved to consider (12) how this could be developed to allow the finite element method to be used to provide estimates of the stress intensity factor (SIF) used to estimate crack growth rate. The SIF is a measure of the amplitude of the dominant singularity at the ends of a crack and can therefore be computed by Morley’s method deployed with a finite element model. After the initial publication of this ground-breaking work, Morley left further development of the method to P. Bartholomew (Bartholomew 1978) while he returned to considering the solution of cylindrical shell problems but now in the context of the finite element method.

He soon realized that the finite element analysis of thin-walled shells is not as straightforward as the corresponding analysis for flat plates. It was clear to him that no work had been done to examine the nature of the mathematical solutions for loaded thin-walled cylindrical shells, which could then be compared with those delivered by finite element models. In developing the Morley shell theory, Morley had implicitly employed the fact that first-order thin-shell theory admits small differences in the physical quantities of the same order as those due to the inherently neglected transverse normal and shearing strains, a conjecture rigorously demonstrated by Koiter (1969). In a paper (11) Morley re-derived his governing partial differential equation for cylindrical shells on a consistent basis within the context of Koiter’s first-approximation theory. He then derived a complete set of complementary function solutions in terms of low-degree polynomials in the shell surface coordinates. These provided a basis for evaluating finite elements that use surface coordinates. This work was developed in conjunction with A. J. Morris (14) to explore the situation in which smooth distributions of displacements give rise to inextensional deformations. These solutions were termed ‘sensitive solutions’ because many finite element formulations produced extraordinarily inaccurate results when attempting to recover these fundamental solutions. The root cause for this problem is an element’s inability to capture stress states induced by inextensional behaviour without also introducing artificial stresses under rigid-body movement.

Morley further explored solutions for cylindrical thin shells in a major paper (13) published in 1976. In this paper he clearly indentified the set of solution types required to cover all the deformation states for finite elements purporting to solve this class of problem. The
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set includes membrane actions, inextensional deformations and some edge effects. He then developed a triangular finite element based on the Fraeijs de Veubeke and Allman flat-plate elements to create a hybrid shell element. This employed polynomial approximations to stress resultants on the interior of the element and a line distribution of normal displacements along the element boundaries to act as Lagrange multipliers to ensure inter-element stress continuity. This approach led to the requirement for mid-side connection quantities with its ensuing problems. Allman, working in the same group at the RAE as Morley, had proposed a procedure for avoiding the problems of mid-side nodes. Morley asserted that this was too complicated to be applied to shell elements so he conducted numerical experiments to find suitable trial functions through which he could eliminate mid-side connection quantities. This allowed him to develop a triangular shell element that could adequately represent the required solution set.

Morley continued his study of the equations of first-approximation shell theory in his paper published in 1982 (15). There he developed solutions for inextensional bending of thin shells but focused on solutions for a class of elements that could have positive, zero or negative Gaussian curvature. The paper presents closed-form exact polynomial solutions to the inextensional bending problem and represents a significant improvement on Flügge (1960), which pertained to spherical shells only. In essence, achieving exact solutions could only be obtained for slowly varying curvature, as would occur in shallow shell elements. However, such a characteristic would be typical of the elements normally employed in the finite element analysis of thin shells. In a subsequent paper (16) Morley pointed out that his inextensional bending solutions could be applied to homogeneous membrane actions from Gol'denveizer’s static-geometric analogue. He then generated a set of simple solutions that could be used as criteria for assessing finite elements for shells of revolution, non-circular cylindrical cylinders and cones.

The role of bending in the finite element analysis of thin shells was addressed in depth by Morley working with M. Mould, and the results of this work were reported in a seminal paper (17) and developed subsequently (19). In undertaking their investigation a new simple flat triangular element was devised using a combined constant strain element (essentially the Turner, Clough, Martin & Topp element (Turner et al. 1956)) and Morley’s constant-bending-moment element. This combined element was termed a ‘vehicle element’ and had displacement connectors at each vertex and a single rotation connector at each mid-side. A ‘transitional element’ was then generated by giving the element a vanishingly small flexural rigidity, which was then further degenerated to form a ‘membrane element’ by removing the mid-side rotation connector. This was a very versatile combination of elements allowing Morley and Mould to examine a comprehensive set of solutions covering membrane states, inextensional bending, edge effects and rigid-body movement. The work revealed two quite different roles for bending freedoms. One concerned inextensional bending moments extending over a whole shell model. The other concerned local rotational movements accompanying the curvature changes of inextensional bending and edge effects. The paper presented extensive numerical comparisons with solutions obtained from classical first-order thin-shell theory. Morley did not feel this work provided the definitive treatment of thin-shell finite elements: the paper contained the statement

In this attempt to promote a better understanding of the finite element method in its application to the linear theory of thin shells it has become clear that much opportunity remains for further work both in the method of assessment and in the field of mathematical abstraction.
It is also worth pointing out that the ‘vehicle element’ devised for this study was itself extremely effective in the analysis of thin-shell problems.

Morley’s researches gave stimulating insights into the fundamentals of finite element analysis in its application to thin-walled shells. This highlighted the difficulty of applying the finite element method in the solution of thin-shell analysis problems. It is unfortunate that the commercial suppliers of modern finite element software packages and their users consider that the use of three-dimensional elements relieves them of any obligation to take into account Morley’s research. This introduces a degree of unreliability into finite element solutions of real-world thin-shell structures; the analysis of thin-shell structures is not a situation in which ignorance is bliss.

In working on the development of finite elements for curved shells Morley formulated the defining equations using tensor calculus and then transformed the tensor formulation into what have been termed physical components. This involves the unwieldy and often daunting task of interpreting the components of general vector and tensor equations referred to curvilinear coordinates. Morley observed that the commonly used concept of physical components, introduced by Truesdell (1953), is useful, but these are referred to tensor coordinate directions and consequently they relate directly to measurable quantities only in the coordinate directions and only where the coordinates are orthogonal. Morley recognized that this represented a deficiency and for finite element applications for curved structures it required measurable quantities that had orthogonal components oriented with respect to the tensor coordinates, whether these are orthogonal or oblique. He therefore returned to considering a method for deriving such measurable quantities based on employing anholonomic coordinates and termed these ‘practical’ components (18). He did not regard this paper as the final word, commenting that the method described in his paper was intended solely as an illustration.

We may conclude this review of Morley’s research and publications by noting that his life’s work has coherence and represents a series of steps in the development of methods for the solution of thin-plate and thin-shell problems. He had a clear mind and a way of probing into the mathematical and physical depths of the problems he addressed that produced some outstanding and significant results.

Morley had a writing style that was both elegant and precise. He adopted a policy of placing drafts of his written papers in his desk and leaving them for several weeks. After this, the papers were carefully read with a fresh eye and then redrafted to enhance clarity. He had a very disciplined approach to his writing, deleting any word not deemed ‘absolutely essential’; this is a trait he passed on to those of us who had the privilege of working with him.

His contribution to his field of research was formally recognized by the award of Cranfield University’s first DSc degree, by his election as a Fellow of the Royal Aerospace Society (1962), the Institute for Mathematics and its Applications (1964), the Royal Academy of Engineering (1982) and the Royal Society (1992).

**PERSONAL LIFE**

Les’s background as an apprentice tool room fitter equipped him with many useful practical engineering skills. As a young man in the 1940s Les had developed a great interest in and owned several classic British motorcycles of the time, including a Rudge, a Chater-Lea, a
Triumph Speed Twin, a BSA Star Twin and—the pride of his collection—a Vincent HRD Comet (figure 1). He was a competent mechanic and enjoyed maintaining and servicing his motorcycles.

Les met and married Norma Baker in 1951 in Bristol. When he took up his appointment at the RAE they set about looking at the possibilities of building their own home. This was realized in 1953, when Les became a leading instigator in setting up the RAE and Farnborough Self Build Association. It involved 25 employees from the RAE, all fellow scientists, engineers and technicians, building 25 houses and bungalows of traditional design in their spare time outside work hours. This was a considerable undertaking for all concerned, and Les saw the scheme through to completion, taking some six years of hard work. As none were time-served building craftsmen, each man acquired and mastered a building skill. Les specialized in plastering, together with floor and wall tiling. Over subsequent years he put his building skills to good use on his own home with a succession of projects. This included extensive landscaping of the garden and the building of a substantial extension to his garage.

During the 1950s Les’s family was growing, with the arrival of Sydney (1952), Peter (1955) and Sally (1958), so his passion for motorcycles necessarily moved to the practicalities of car ownership, in particular Volvos, which never went near a commercial garage: he carried out all his own automobile servicing and repair work, including major engine overhauls. His practical mechanical engineering skills enabled him to design and build many specialized

Figure 1. Les on his Vincent HRD Comet circa 1945.
tools, which proved invaluable. These ranged from heavy-duty engine hoists to intricate handmade tools for detailed work.

Les was very much a family man, and he and Norma would take their three children on continental camping holidays. He would take most of his annual leave in one ‘lump’, enabling the family to enjoy a long holiday during the summer months of the 1960s and 1970s. He would drive considerable distances, and memorable family camping holidays were spent visiting most countries of Western Europe.

Throughout his working life Les was generous with his advice and time to colleagues and students alike. His enthusiasm for his research work continued throughout his retirement. In later life he sometimes reflected on his humble educational origins and would often remark to his family how lucky he had been to enjoy a long career, both at work and in retirement, in a subject he loved.

Les continued with his research into his eighties but his failing health made this task increasingly difficult. He quietly accepted that he could no longer apply the academic rigour that his work demanded. Instead, Les turned his attention to enjoying the outdoor life and looked forward to walks with members of the family and the opportunity to reminisce about his childhood and past times. Norma supported Les at home as his health continued to decline. He died on 16 June 2011 aged 87 years.

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REFERENCES TO OTHER AUTHORS


The following publications are those referred to directly in the text. A more complete bibliography is available as electronic supplementary material at http://dx.doi.org/10.1098/rsbm.2015.0029 or via http://rsbm.royalsocietypublishing.org.