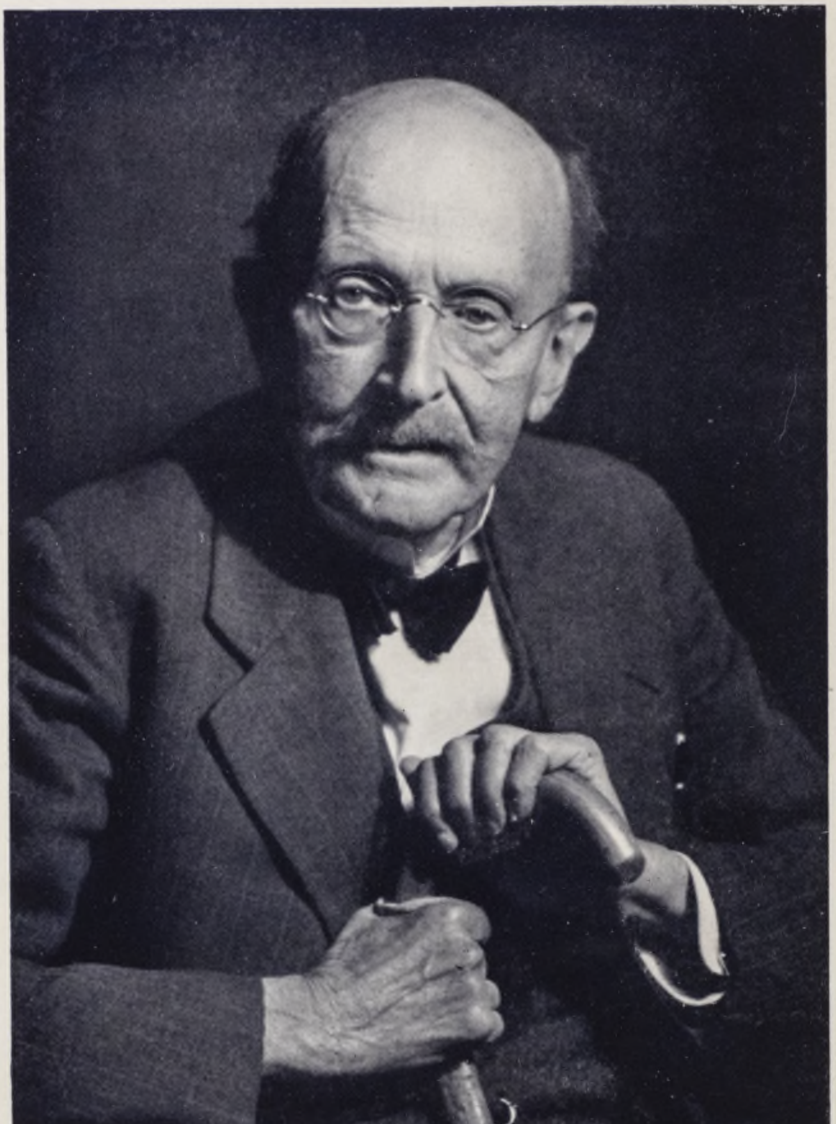

M. Planck

MAX KARL ERNST LUDWIG PLANCK

1858–1947

Almost half a century has elapsed since Max Planck's discovery of the quantum of action, a time sufficiently long to estimate its importance for science and, more generally, for the development of human thought. There is no doubt that it was an event of the first order, comparable with the scientific revolutions brought about by Galileo and Newton, Faraday and Maxwell. Like these it has changed the whole aspect of physics and deeply influenced all neighbouring sciences, from chemistry to biology. Its philosophical implications reach far beyond the epistemology of science itself into the deepest roots of metaphysics.

What kind of man was he who initiated this great movement? Apart from his numerous works, papers and books, we have a short *Scientific Autobiography* which is a great help in the understanding of his motives and his reactions. There are, furthermore, a series of articles published in *Naturwissenschaften* on the occasion of Planck's sixtieth birthday, amongst them an excellent biographical one by Sommerfeld. All this valuable material will be used and quoted in the following attempt to give a picture of Planck's personality. Yet my best help must be the memory of years of personal contact and friendship, which have left an unforgettable impression.

Planck came from an old family of lawyers, public servants and scholars. One of his ancestors was a minister in Suabia who later became a professor of Divinity at Göttingen. One of the grandchildren of this man was to become a celebrated jurist, Professor of Law at Göttingen, distinguished as the founder of the German Civil Code (Deutsches Bürgerliches Gesetzbuch). The Planck-Strasse, where I later lived for several years, was called after him. He lost his sight at an early age, and I remember the venerable figure of the blind old 'Excellency' from my own student days. He and Max Planck's father were cousins; the latter also a distinguished jurist and Professor of Law at Kiel University. In 1867 he was called to Munich, and is said to have enjoyed the confidence of his colleagues and played an important part in the administration of the University.

This ancestry of excellent, reliable, incorruptible, idealistic and generous men, devoted to the service of the Church and State, must be remembered if one wishes to understand the character of Max Planck and the roots of his success. For his work was directed by just the same traits combined with a sincere belief in the simplicity of nature and an absolute confidence in logical reasoning from facts.

Max Karl Ernst Ludwig Planck was born at Kiel on 23 April 1858. Here he spent his early childhood. When he was nine years old the family moved to Munich. He became a pupil of the Maximilian Gymnasium (Grammar School), where he received his first scientific inspiration from his mathematics teacher, Hermann Müller, an ingenious and sharp-witted man who knew how to demonstrate the laws of physics from simple, forceful examples.

In his autobiography Planck says about these beginnings: 'What led me to my science and from my youth filled me with enthusiasm, is the fact—not at all self-evident—that our laws of thinking conform with the lawfulness in the passage of impressions which we receive from the outer world, thus making it possible for man to gain information about that lawfulness by mere thinking. In this it is of the highest significance that the outer world represents something independent of us and absolute with which we are confronted, and the search for the laws which govern this absolute has appeared to me as the most fascinating work of a lifetime.'

These ideas are characteristic of Planck's whole attitude to science, and it was Müller who encouraged and assisted him in developing them.

The principle of conservation of energy was welcomed by Planck 'like a gospel' as the first of those 'absolute' laws.

When it came to the choice of a profession there were, however, other competing interests. He considered for a time the study of classical philology. He tried his musical gifts in composition, but came to the conviction that they did not suffice for original production. In the end physics prevailed. Music, however, remained an essential part of his life. He became an excellent pianist and found in playing deep enjoyment and recreation.

Planck studied for three years at the University of Munich. There were no chairs of theoretical physics at that time, so he attended the lectures on mathematics by Gustav Bauer and Ludwig Seidel, and on physics by Ph. von Jolly. They gave him a solid foundation of knowledge, but the wide horizon of science was opened to him only when he went to Berlin, where he studied one year under Helmholtz and Kirchhoff. However, so he reports, it was not from their lectures that he benefited. Helmholtz was never properly prepared, he improvised with the help of a little notebook and made mistakes in the calculations on the blackboard, so that his students felt that he was just as bored as they themselves. Kirchhoff's lectures on the other hand, were carefully worked out, each sentence well considered, but the whole was dry and monotonous. It was when Planck turned to their writings that he was fascinated. His main interest was still the principle of conservation of energy. Soon he discovered a new source of enlightenment in the publications of Clausius, which made a deep impression on him through their clear language and lucid explanations. Here he learned for the first time to distinguish between the two fundamental theorems, as formulated by Clausius, and from this time on his whole scientific thinking was rooted in the conceptions of thermodynamics.

His doctor's thesis, Munich 1879, is the first of his papers dealing with

the second theorem. He was not satisfied with Clausius' definition of irreversibility, namely that a process is irreversible if it cannot be made to go in the opposite direction, like the conduction of heat; Planck regards this as insufficient, as it does not exclude the possibility of reversing the result of the process in an indirect way, which is just what should be excluded. So he suggests that a process should be called irreversible, or as he prefers to say 'natural', if it cannot be completely undone without compensation. Entropy is a measure of the 'predilection' of nature for the final state and it increases in all 'natural' processes.

Planck's expectation of a favourable reception of his paper was not fulfilled. Helmholtz was indifferent, Kirchhoff objected that entropy was only measurable by reversible processes and should therefore not be applied to irreversible ones. An attempt to get in personal contact with Clausius in Bonn failed, and a correspondence with Carl Neumann in Leipzig remained futile.

But Planck was not discouraged. He continued his work on thermodynamics in a series of papers from 1880 to 1892, the first of which was used as thesis for the admission as Privatdozent ('Habilitation') at Munich University, which he obtained in 1880.

If one reads these articles now, one has the feeling of meeting old acquaintances. Everything seems to be familiar, the definitions, demonstrations, even the symbols. The reason is that we have all been nursed with Planck's book on thermodynamics, in which he has condensed the result of his previous work. So systematic was his mind, so well considered every word, every formula he wrote, that hardly anything had to be changed in the final compilation. This book *Vorlesungen über Thermodynamik* appeared first in 1897 and has since had numerous new editions.

Planck's interest in physics had always a philosophical background, namely, his fundamental belief formulated above, that the human mind can penetrate into the mysteries of nature by pure thinking because of a harmony between the laws of mind and the laws of nature. Therefore, he always preferred deductive, sometimes even axiomatic methods. This is quite obvious in his thermodynamical work. Once he was convinced of the general and universal law of the increase of entropy he tried to deduce from it as much as possible. Equilibria are then characterized by maxima of entropy, or by equivalent extrema of other thermodynamical potentials. These extremal principles of thermodynamics formed therefore the basis of his work, in contrast to the usual methods of the physico-chemists who preferred special cyclic processes which appealed to their intuition. Planck did not know at the time that his extremal principles had already been discovered and applied by Willard Gibbs, and it is only natural that he felt a certain disappointment when he found this out. He was still Privatdozent at the University of Munich and waited with some impatience for the offer of a professorial chair. Yet the chances were slight since theoretical physics was not an acknowledged academic subject.

In order to make himself better known in the scientific world Planck decided to compete for the prize for 1887 of the Philosophical Faculty of Göttingen,

demanding a thesis on the conception of energy. Before this paper was finished Planck was offered a chair as 'extraordinary' (i.e. not full, or 'ordinary') Professor of Theoretical Physics at the University of Kiel, the place of his birth. He has told us in his autobiography that the day when he received this call was one of the happiest of his life; for although he was living quite comfortably in the house of his parents, he yearned for independence and a house of his own. Now it became his ambition to justify the confidence shown to him by his colleagues in Kiel. He quickly finished his work for Göttingen and was successful against two competitors, though he was awarded only the second prize. The report of the Faculty contains a paragraph criticizing his attitude to Weber's law of electro-dynamic interaction. Wilhelm Weber, then Professor of Physics at Göttingen, was involved in a sharp controversy with Helmholtz. If Planck's siding with the latter cost him the first prize at Göttingen, he was soon compensated by the interest which the Berlin physicists took in the young scholar.

Planck now returned to his favourite subject and wrote four big papers with the common title 'On the principle of the increase of entropy' (1887, 1891). The first of these, where again instead of 'irreversible' the term 'natural' processes is used, contains the introduction of thermodynamical potentials and the derivation of their extremal properties, already mentioned above. Planck calls the expression $w=u+pv-Ts$ 'Massieu's function' which to-day goes under the name of Gibbs' Potential. The equilibrium of different phases is discussed and a general outline of the theory of chemical equilibrium given. The following papers fill this frame with detail. For obtaining concrete results explicit expressions for the thermodynamical potentials must be known. Planck made simple and natural assumptions, for instance that in a dilute solution the thermodynamic potential is a linear function of the concentration of the dissolved particles. In this way he developed a formal apparatus which he applied to deducing observable facts.

This was a period when the newly established science of physical chemistry produced discoveries and theories in abundance. The law of mass action was established by Guldberg and Waage; the properties of dilute solutions were studied by Van't Hoff and those of electrolytes by Arrhenius. Planck deduced many of these results from his principles, sometimes independently of and even prior to the chemists. He gave a thermodynamical derivation of the dissociation of gases, of the osmotic pressure and of the lowering of the freezing point in solutions. Discussing the observed values of the freezing point in many solutions of salts, he arrived at the conclusion that the salts in solution must be dissociated. He saw in this result a thermodynamical foundation of the theory of electrolytic dissociation which Svante Arrhenius had developed about the same time from a large amount of experimental material. Arrhenius however rejected Planck's thermodynamical reasoning because he believed that the ionic state was essential for his hypothesis. Planck insisted that the thermodynamical laws are equally applicable to charged and neutral particles—which is certainly right; yet the actual form of the laws may well

depend on the charge, as modern investigations have shown (Debye and Hückel). To-day we can therefore say that none of the adversaries was quite right. If Planck resented the misunderstanding of his work there is no trace of it in his publications. He welcomes the agreement of conclusions reached by so widely different methods and sees in it a confirmation of his belief in the fundamental character of the second law of thermodynamics.

After Kirchhoff's death the Philosophical Faculty of Berlin, apparently under Helmholtz's influence, offered in 1899 the Chair of Theoretical Physics to Planck. It was an 'extraordinary' Professorship which however in 1892 was converted into an 'ordinary' one. Planck describes the subsequent years as a most important period which widened his scientific horizon by bringing him, for the first time in his life, in personal contact with leading men in his field. Helmholtz, whose works he had admired, won also his personal veneration by his simplicity, dignity and kindness. A word of appreciation from him made him happy. Planck was also on excellent terms with Helmholtz' successor, A. Kundt, and a close friendship developed later with H. Rubens.

In the first period of Planck's life in Berlin he suspended his thermodynamical work for another task, which attracted his musical interest.

A big harmonium with numerous keys, built on Helmholtz' suggestion in pure tuning, was at that time delivered to the Department of Physics. Planck learned to play this complicated instrument and studied the effect of the pure tuning as compared with the tempered one introduced by Bach; he found the unexpected result, published in a special paper (1893), that our ear prefers decidedly the tempered scales.

At that time there had grown, under the leadership of Wilhelm Ostwald, a school of 'energetics' which proclaimed that the law of energy was a sufficient basis for the derivation of the whole of physics and chemistry. Boltzmann took up this challenge and was soon involved in a sharp controversy with this group. Planck came to his assistance in an article (1896) which revealed, for the first time, his polemic gifts. Ostwald distinguished three different types of energy corresponding to the three dimensions of space: energy of distance, of surface and of volume. Planck replied that there are cases where no volume energy in Ostwald's sense exists, as for instance in the case of an ideal gas where the energy depends only on temperature and not at all on the volume. Another point of controversy was the failure of the energetics school to understand Clausius' Second Theorem. They compared the flow of energy from a higher level of temperature to a lower with the falling of a weight without taking into account the irreversibility of the process. This superficial analogy was violently opposed by Planck. Although the principle of conservation of energy was, right from the beginning, foremost in his mind, he was perfectly clear that it alone was an insufficient foundation on which to build up mechanics and that a much more powerful principle, such as that of least action, was needed. With regard to thermodynamics he defended Clausius' distinction between reversible and irreversible processes.

Planck complains in his autobiography that in this case, as in many others,

he did not succeed in convincing his colleagues by arguments which seemed to him, though theoretical, perfectly valid. In fact, the defeat of the energetics' school was eventually due to Boltzmann's atomistic theory which Planck, at that time, did not fully appreciate.

Boltzmann's investigations on the kinetic theory of gases had led him to the construction of a certain quantity H, depending on the velocity distribution of the molecules, which he could prove was continuously decreasing in time. By identifying $-H$ with the entropy he obtained a kinetic interpretation of Clausius' second law. This atomistic conception of irreversibility made a profound impression and was generally accepted.

Planck confesses himself that to begin with he was not only indifferent but somewhat doubtful about Boltzmann's statistical views. The reason is that he believed the law of the increase of entropy to be just as general and free from exception as the law of conservation of energy while in Boltzmann's theory it appeared only as a probable law: the quantity H might occasionally increase, the entropy decrease.

E. Zermelo, a young and temperamental pupil of Planck, attacked Boltzmann's statistical ideas, using a theorem of Poincaré according to which any mechanical system is quasi-periodic; how then could a quantity defined in terms of mechanical variables, like Boltzmann's H, permanently decrease? It was not difficult for Boltzmann to refute this argument by showing that the definition of his function H involved probability and that therefore the theorem of its decrease had to be understood statistically. The controversy was carried on from both sides with considerable heat; Boltzmann brought into play his sarcastic wit, hitting also Planck himself who had backed his pupil. From that time on the relation between the two men was not too friendly, until the atomistic derivation of the radiation law by Planck mellowed Boltzmann's mind. In fact, nobody has done more to foster and spread Boltzmann's ideas than Planck; his theory of radiation is completely built on them and modelled on their analogy, and one of its main results was the determination from radiation data, of the constant k in Boltzmann's fundamental relation $S=k \log P$ between entropy S and probability P.

Discussing this question in his autobiography Planck expresses a slight resentment at the usual nomenclature 'Boltzmann's constant' for this factor k, pointing out that Boltzmann had neither introduced it nor ever thought of determining it numerically, leaving this task to his colleague Loschmidt. This indicates that the quarrel with Boltzmann had left traces in his mind which still were discernible in his old age when he wrote those lines.

Planck's interest in heat radiation was roused by the experimental work done at the Physikalisch-Technische Reichsanstalt (National Physical Laboratory) in Berlin-Charlottenburg on the spectral distribution of the radiation emitted by a 'black body'. There were two prominent teams at work, Lummer and Pringsheim, Rubens and Kurlbaum. Their measurements directed Planck's attention to Kirchhoff's theoretical investigations of the properties of the radiation of a 'black body', that is to say, the radiation in a cavity bounded by

perfectly reflecting walls and containing arbitrary emitting and absorbing substances. He had shown that an equilibrium is established in the course of time where all substances have the same temperature and the radiation in all its properties, including the spectral distribution (energy per unit wave-length), is independent of the bodies and only a function of temperature. This so-called 'normal spectrum' is therefore something 'absolute', a great attraction for Planck, whose philosophical mind was always directed towards the search for the 'absolute'. Henceforward the explanation of this law was his aim which he pursued from 1896 on with amazing persistency, always in contact with the parallel experimental investigations of the Reichsanstalt. The series of papers (1897–1901) exclusively dealing with this problem and ending with complete success are a testimony not only to Planck's skill and ingenuity, but also to his character, his unbending will and untiring industry, his cautious patience combined with greatest audacity. His was, by nature, a conservative mind; he had nothing of the revolutionary and was thoroughly sceptical about speculations. Yet his belief in the compelling force of logical reasoning from facts was so strong that he did not flinch from announcing the most revolutionary idea which ever has shaken physics.

Maxwell's electromagnetic theory of light was at that time beginning to conquer the continent. Planck accepted and used it for his purpose. As according to Kirchhoff the nature of the emitting and absorbing substances was irrelevant for black body radiation, Planck chose a simple model, namely linear oscillators with different proper frequencies and small damping. He expected to find that the exchange of energy by emission and absorption of radiation would lead automatically to a final equilibrium state in agreement with Kirchhoff's results. The first step in this direction consisted in the calculation of the averaged emission and absorption of an oscillator situated in a given radiation field. To represent the latter, the electromagnetic field components were expanded in Fourier series with arbitrary amplitudes and phases. This is now a standard method of theoretical physics and so well known that few physicists will realize the effort needed to invent it. This effort is still discernible in Planck's book on the theory of radiation which appeared much later and contains a condensed form of the calculations. The main result was a relation between the mean energy u of an oscillator of a given frequency ν and the mean energy density ρ of the surrounding radiation in stationary (statistical) equilibrium, $\rho = \frac{8\pi\nu^2}{c^3} u$. This relation is independent of the damping of the oscillator, a fact which meant a considerable simplification of the problem; for it was thus reduced to the study of the system of oscillators, each of which had only one degree of freedom. On the other hand, Planck's original hope, that the oscillators would produce an exchange of energy between different frequencies and thus lead directly to the establishment of the normal spectrum, was disappointed, as each oscillator was found to be sensitive only to the radiation of its own frequency.

Here again a controversy with Boltzmann developed. The latter denied that

the interaction of the oscillators with the radiation was irreversible and pointed out that every single process considered by Planck could just as well go in the opposite direction, even the emission of a spherical wave by an oscillator; for in a stationary state to each expanding wave there corresponds a contracting wave which transfers energy to the oscillator. This is formally correct, but nevertheless Planck was perfectly right and showed a deeper insight in a matter of statistical physics even than Boltzmann. Just as in a gas the mechanical reversibility can only be transformed into thermodynamical irreversibility through the introduction of the hypothesis of molecular disorder (i.e. by replacing rigorous expressions by averaged ones) one has in the case of radiation to introduce a corresponding assumption which Planck called the hypothesis of 'natural radiation'. It consists in averaging the phases and amplitudes of the simple harmonic waves into which the radiation can be decomposed. Thus this dispute was not futile but led Planck to greater clarity about his own procedure.

After the failure of his first attempt Planck looked for another way of attack and found it in the use of thermodynamical conceptions and finally in the application of the statistical methods due to his adversary Boltzmann.

Planck had the idea that he would obtain simple results by investigating the relation of energy U to entropy S, not to temperature T. For a system in a fixed volume one has the thermodynamical formula $TdS=dU$, from which one easily obtains

$$\frac{d^2S}{dU^2}=\frac{-1}{T^2\frac{dU}{dT}}$$

If the energy U is a known function of the temperature, the right-hand side can be regarded as a given function of U; hence one has a differential equation to determine $S(U)$.

Now at that time W. Wien had published a law for the spectral distribution of radiation of the form $U(T)=Ae^{-B/T}$ (B proportional to the frequency), which was attractive by reason of its similarity to Boltzmann's statistical distribution law and also well confirmed by experiment in a wide spectral region. If it is introduced into the previous formula one finds

$$\frac{d^2S}{dU^2}=-\frac{1}{BU}.$$

a result so surprisingly simple that Planck first believed it would be generally correct. At this point Planck's close connexion with the experimentalists of the Reichsanstalt was decisive. Through the measurements of Lummer and Pringsheim and still more those of Rubens and Kurlbaum, it became more and more clear, that Wien's radiation law, though very satisfactory for short waves and low temperatures, was not in agreement with the facts for long waves and high temperatures, where it had to be replaced by another law,

namely that the energy per frequency interval is proportional to the temperature, $U(T)=CT$. This law is now known under the name of Rayleigh-Jeans; in fact Lord Rayleigh showed about the same time, in 1900, that it is a necessary consequence of ordinary statistical mechanics applied to radiation, and this point of view was stressed by Jeans again in 1909. One finds in this case

$$\frac{d^2S}{dU^2}=-\frac{C}{U^2},$$

again a surprisingly simple result.

Thus Planck had two limiting cases to ponder about. In describing this period Planck says that fate was kind to him. He had often been pained by the lack of interest of his colleagues in his work; but now this turned to his advantage; nobody else had the idea to consider the entropy as the crucial quantity, and he was allowed to follow his plan to its end without interference or competition. The next problem was to combine the two expressions into one, that they appear as the limiting cases for large and for small U. Planck noticed at once that this is achieved by taking the reciprocal of d^2S/dU^2 and adding the two expressions $-BU$ and $-U^2/C$; taking again the reciprocal he found the differential equation

$$\frac{d^2S}{dU^2}=\frac{-C}{U(U+BC)}.$$

This adding up was one of the most fateful and significant interpolations ever made in the history of physics; it reveals an almost uncanny physical intuition. Five years later it became much more comprehensible and natural by an interpretation due to Einstein (made in the same paper where he correlated Planck's quanta with the photoelectric effect); he remarked that the reciprocal of d^2S/dU^2 has a simple physical meaning: it represents the mean square fluctuation of energy $\overline{\Delta U^2}$; and it is well known that mean square fluctuations are additive, if due to independent causes. This argument was then used by Einstein as an indication of the independent existence of light quanta; but that is beyond the scope of this article.

The combined formula, which contains two constants, can now be integrated and leads directly to the new radiation formula, which Planck submitted to the Berliner Physikalische Gesellschaft on 19 October 1900.

He tells us that the next morning his colleague Rubens appeared to inform him that in the same night, after the meeting, he had compared Planck's formula with his own measurements and found everywhere satisfactory agreement. Lummer and Pringsheim believed first that there were deviations but discovered soon that these were due to an error in computation. Many later experiments have been made to check Planck's formula, with the result that the agreement has been found to become more and more perfect with the improvement of methods of measurement.

Yet it was only an interpolation, a real physical meaning had to be found. At this point Planck's attention was directed to Boltzmann's fundamental relation between entropy and probability, $S=k \log P$. Hence he investigated the question whether the expression of P obtained by substituting for S the value corresponding to the new radiation law could be interpreted as a probability. In a lecture given to the German Physical Society on 14 December 1900, he announced the result that this interpretation is possible indeed. Apart from the constant k which was recognized to be the absolute gas constant per gram-molecule, there appeared a new constant of the dimensions (energy $\times$ time) which he called the 'elementary quantum of action' and denoted by h, now always quoted as Planck's constant. Planck, right from the beginning, saw the essential feature of his discovery in this 'quantum of action'. His contemporaries were more stirred by the 'quantum of energy' $\epsilon_0=h\nu$, and by Planck's contention that the energy of the emitting and absorbing oscillators was 'atomistic', always a multiple of ϵ_0. It was this assumption which lead Planck to the expression for the mean energy of a system of oscillators; using Boltzmann's distribution law one has

$$u=\frac{\sum_{n=1}^{\infty} n\epsilon_0 e^{-n\epsilon_0/kT}}{\sum_{n=0}^{\infty} e^{-n\epsilon_0/kT}} = \frac{\epsilon_0}{e^{\epsilon_0/kT}-1}, \quad \epsilon_0=h\nu,$$

and, with the help of Planck's previous result concerning the relation of radiation and oscillators, one finds the expression for the radiation density

$$\rho=\frac{8\pi\nu^2}{c^3}u = \frac{\frac{8\pi h}{c^3}\nu^3}{e^{h\nu/kT}-1}.$$

This formula contains all previously known radiation laws, the law of Stephan and Boltzmann for the total radiation, Wien's displacement law and, of course, the two limiting laws of Rayleigh-Jeans for large T and of Wien for small T. From the known constants of these Planck derived numerical values for his two constants k and h. From k he calculated the number N of atoms per gramme molecule (Avogadro's or Loschmidt's number), and, with the help of Faraday's law, the elementary electric charge e; his values were much more reliable than all known before and were later confirmed by many other methods.

Planck was perfectly clear about the importance of his discovery. We have not only the testimony of his wife but also an account of his son Erwin, given to and reported by Professor Bavink. It was in 1900 when his father, on a walk in the Grunewald, near Berlin, said to him: 'To-day I have made a discovery as important as that of Newton'. Planck has, of course, never said anything like that in public. His modest and reluctant way of speaking about his work has caused the impression that he did himself not quite believe in his result. Therefore, the opinion spread, especially outside Germany, that Planck 'did not seem to know what he had done when he did it', that he did

not realize the range of his discovery. That this is wrong can clearly be seen from his autobiography; though it was written in his old age, we have no reason to doubt that it correctly reflects his thoughts in the years following his discovery. Planck reports that he tried hard to fit the quantum of action into the frame of classical theory, but with no success. Then he continues: 'But this quantity (the constant *h*) proved to be unwieldy and resistive against all attempts of this kind. As long as it could be regarded as infinitely small, i.e. for larger energies and longer periods, everything was in good order. But in the general case there appeared somewhere a cleavage which became the more conspicuous, the faster the vibrations considered. The failure of all attempts to bridge this gulf, soon removed all doubt that the quantum of action plays a fundamental part in atomic physics, and that with its appearance a new epoch of physical science has begun. For it forebodes something unheard-of destined to reform thoroughly our physical thinking which since the invention of the infinitesimal calculus through Leibniz and Newton was based on the assumption of continuity of all causal relations.'

It has been generally acknowledged that the year 1900 of Planck's discovery marks indeed the beginning of a new epoch in physics. Yet during the first years of the new century very little happened. It was the time of my own student days, and I remember that Planck's idea was hardly mentioned in our lectures, and if so as a kind of preliminary 'working hypothesis' which ought of course to be eliminated. Planck himself turned to other fields of work. But that he never forgot his quanta is shown by the publication, in 1906, of his book *Vorlesungen über die Theorie der Wärmestrahlung* which made a profound impression by the masterly presentation of the successive steps which led to the quantum hypothesis.

A year earlier Einstein's paper, already quoted, had appeared in that famous volume (1905) of *Annalen der Physik*, which contains also two other fundamental articles of Einstein, one on relativity and one on the Brownian movement. Einstein showed that the quanta were not a feature of radiating heat, but of radiation in general, and he produced experimental and theoretical evidence for a corpuscular interpretation of light. A series of phenomena like the photo-electric effect, the excitation of X-rays by the impact of electrons on the target, Stokes' rule of fluorence could be simply explained in terms of 'light quanta' $h\nu$. Now the interest of the experimentalists was roused and progress became quicker. From the standpoint of the theory a decisive step was made again by Einstein in 1907, when he applied Planck's formula for the mean energy u of a system of oscillators (given above) to the vibrations of atoms, molecules and solids, explaining in particular the deviations of the specific heat of solids from the classical law of Dulong and Petit. This initiated a great amount of experimental research, for instance the investigations of Nernst and his school on the specific heat at very low temperatures. But I cannot follow up the history of quantum theory in general as this would mean a description of the greatest part of modern physics; I must confine myself to Planck's own contribution.

There is a publication of his in 1910 where he summarizes the situation. He discusses a number of papers by J. J. Thomson, Larmor, Stark and Einstein which use the quantum hypothesis for explaining divers phenomena; but he is very cautious in regard to Einstein's revival of the corpuscular theory of light. Its main argument is the existence of electrostatic fields which from Einstein's standpoint would be something completely different from radiation fields. Can one abandon the unification due to Maxwell of all electromagnetic fields in view of the existing evidence? His conclusion is that electrodynamics is very probably right, but physical statistics possibly wrong.

In his lecture to the Solvay Congress 1911, he made a decided attempt to develop a modified statistical mechanics by assuming the phase space of Gibbs divided up in finite cells of the size h for each pair of conjugate variables p, q. At the same time he changed his assumption about absorption and emission; absorption was supposed to be continuous, emission discontinuous (1911). This strange hypothesis seemed to him the only way out of the dilemma between quantum effects and electro-magnetic theory. Many physicists, particularly those of the younger generation, regarded Planck's 'second quantum theory' as a weak compromise. It is to-day hardly worthwhile to discuss its pro and contra. But one must not forget that it led to a most important result, the zero point energy $\frac{1}{2}h\nu$ per oscillator. It appears formally when one expands Planck's formula for the resonator energy for large temperatures

$$u = \frac{\epsilon_0}{e^{\epsilon_0/kT} - 1} = kT - \frac{\epsilon_0}{2} + \ldots ,$$

where the terms indicated by dots vanish for $T \to \infty$. This expression shows that Planck's formula does not precisely tend to the equipartition value kT (which corresponds to Rayleigh-Jeans' formula for ρ) but differs from it by $\frac{1}{2}\epsilon_0 = \frac{1}{2}h\nu$. Planck's new statistics leads to a value for u which is larger than that given above by $\frac{1}{2}\epsilon_0$ and tends therefore exactly towards kT for $T \to \infty$. In this way he found a new approach to Nernst's theorem and the zero point entropy of gases (1916). Later research has demonstrated the reality of the zero point energy, for instance by its influence on the scattering of X-rays in crystals. Planck himself regarded his second quantum hypothesis as so important that he made it the basis of the second edition of his book *Wärmestrahlung*, which appeared in 1913. Another modification of the theory is contained in a series of papers (1915, 1917) where he replaced the oscillators by rotators. Here he used a method first introduced by Einstein in his theory of Brownian motion and later improved and applied to radiation by Fokker; it describes the changes in time and space of a distribution of particles subject to small irregular impulses with the help of a partial differential equation containing as coefficients the mean displacement and the mean square displacement for a given small time. This formula is now generally quoted as the Fokker-Planck equation, and its full range of application seems to be not at all exhausted yet.

The year 1913 marks a turning point in quantum theory as there appeared

Niels Bohr's first papers on the quantum theory of the electronic structure of atoms. A straight development in which Planck took an active part, led from here to modern quantum mechanics. But in the intermediate period he turned his mind to many other subjects of which a short account must be given.

His investigations on radiation convinced him that the electromagnetic field showed statistical features similar to those of a gas; the amplitudes and phases of the elementary waves are arbitrary and may be distributed at random. In this way he came to his theory of 'natural' or 'white' light (1902), which later was taken up by his pupil Max von Laue. Then he became interested in ordinary optics, in particular in Drude's theory of dispersion in which he introduced radiation damping of the oscillators (1902, 1903, 1904, 1905). He calculated the extinction of light in an optically homogeneous medium of normal dispersion and compared his results with an older theory of Lord Rayleigh concerning the propagation of light in a vacuum in which numerous non-conducting particles are dispersed; he found the same law for the extinction coefficient as had Rayleigh, although the dispersion law is completely different for the two models. The experiments made by Hagen and Rubens on the optical properties of metals induced Planck to a theoretical study of this subject (1905).

He returned to the theory of gases (1908) and generalized Boltzmann's method in such a way that it could take into account van der Waal's corrections due to the finite volume of the molecules.

But the subject that caught Planck's imagination more than anything else was Einstein's theory of relativity, published in 1905. In Planck's scientific autobiography is a remarkable page where he explains how his search for 'the absolute', the main spring of his scientific activity, is compatible with his interest in the principle of relativity. 'One might regard this as a contradiction. . . . This presumption is based on a fundamental error. For everything "relative" presupposes something "absolute", it has only significance if it is opposed to some absolute. The often quoted sentence "Everything is relative" is just as misleading as thoughtless. So at the bottom of the so-called theory of relativity there is something absolute, namely the metric of the space-time continuum, and it is just a particularly attractive problem, to discover the Absolute which lends a meaning to a given Relative. . . .' He found the attraction of relativity in the search for those invariants which represent the 'absolute'. The velocity of light which in classical physics has only a relative meaning becomes in relativity an absolute invariant. The next important invariant is the action integral of mechanics; the laws of motion can be obtained by the principle of least action in relativity.

Planck applied this idea first to a mass point (1906) and found the relativistic form of the mechanical equations, a little earlier than Minkowski. He discussed Kaufmann's measurements of the deflexion of β-rays in regard to their bearing on the principle of relativity (1906, 1907). In 1908 he published a long paper on the general dynamics of moving systems, in which he expands the

thesis of his pupil K. v. Mosengeil (published by Planck in 1907 after the young author's untimely death). As relativity teaches that mass is proportional to energy, and as the energy of a body depends on its heat content, a separation of mechanics and thermodynamics is impossible. Planck develops a combined theory based on the relativistic invariance of the principle of least action and obtains the transformation laws for energy, momentum, entropy and temperature; with the help of these the expression of these quantities in terms of the velocity can be obtained from their values in the rest system. If one supposes that these expressions derived for steady motion also hold for acceleration, one can write down the equations of motion. This paper contains a most remarkable section (§18) in which he predicts the possibility of utilization of 'atomic energy'. He is perfectly clear that every body contains, in his rest-mass, a colossal amount of 'latent' energy, and says: 'Though the actual production of such a "radical" process might have appeared extremely small only a decade ago, it is now in the range of the possible, through the discovery of radio-active elements and their transmutation, and in fact the observation of continuous production of heat of radioactive substances is direct evidence for the assumption that the source of this heat is just nothing else than the latent energy of the atoms'.

The Prussian Academy, mainly on the instigation of Planck, Nernst and Haber, created a special chair for Einstein which allowed him to pursue his ideas unhampered by teaching and routine work. Now for many years Planck and Einstein met at regular intervals at the Berlin Academy, and a friendship developed which went far beyond the exchange of scientific ideas. Yet it is difficult to imagine two men of more different attitudes to life: Einstein a citizen of the whole world, little attached to the people around him, independent of the emotional background of the society in which he lived—Planck deeply rooted in the traditions of his family and nation, an ardent patriot, proud of the greatness of German history and consciously Prussian in his attitude to the state. Yet what did all these differences matter in view of what they had in common—the fascinating interest in the secrets of nature, similar philosophical convictions, and a deep love of music. They often played chamber music together, Planck at the piano and Einstein fiddling, both perfectly absorbed and happy. Planck was an excellent pianist and could play on demand almost any piece of classical music, a great many by heart. He also liked to improvise either on a theme given to him, or on old German folk-songs which he dearly loved.

The collaboration of Planck and Einstein made Berlin, in the years preceding the First World War, the greatest centre of theoretical physics in the world. I was also fortunate to be called to Berlin. Planck wished to be freed from a part of his duties in routine teaching and persuaded the Prussian Minister of Education to found a new (extraordinary) Chair at the University of Berlin. This was offered to me, but alas, on the day of mobilization, 2 August 1914. There was not much of teaching and peaceful research for me during the following four years of war, yet I was in Berlin for long periods and saw

Einstein and Planck frequently. A short walk brought me from my own house to Planck's, a villa in the suburb of Grunewald. I remember his studio, the walls covered with books, simple furniture, among which a high desk (like those found in old-fashioned offices) where he used to work standing upright. I had never been his pupil, not even attended one of his courses; I knew his papers and books, I had seen him from a distance at scientific meetings and perhaps exchanged a few words with him. He was at that time already a great and famous man, and I approached him with some shyness. But his kindness, his lovable expression, the hospitality of his house removed very quickly the barrier of age and experience. We had many fascinating discussions on physics and the topics of the day. He had very definite views and expressed them frankly, even if he did not expect agreement, but never in an offending way. The same systematic order, tidiness and clarity which distinguish his writings, were also characteristic of his attitude to the small and big questions of ordinary life. During the years of the war a great change came over him; sorrow darkened his friendly expression. It was not only the general suffering, the catastrophic end of the struggle which hurt his patriotic feeling deeply, but terrible personal loss. Planck's first wife, Marie Merck had died in 1909. He had married again, Marga von Hoesslin. Three of the four children of his first marriage died during the war period. His eldest son Karl was killed in action near Thiaumont, France, in 1916. The two daughters, Emma and Margarete, were twins. One of them married Professor Ferdinand Fehling; she died in 1917 in childbirth; her sister took charge of the orphan baby and later married the widower. A year later exactly the same thing happened to her; she died after her first confinement, while the child lived. Both children were partly educated in the grandfather's house. Only one son, Erwin, of his first wife was left, and a young son, Hermann, of his second marriage. More tragedy was to come.

In spite of all this worry and sorrow Planck continued his scientific work, returning to his long neglected quantum theory, which through Bohr's papers of 1913, had suddenly become the focus of interest in the world of physics. Bohr's method of quantization was extremely successful for the one-electron problem; how could it be generalized for a system of many electrons? This problem was almost simultaneously solved by Sommerfeld, Epstein and Planck (1915, 1916). The methods differ in form but lead in all practical cases to essentially the same results. While Sommerfeld considers multi-periodic systems for which a separation of the Hamiltonian in independent pairs of co-ordinates and momenta is possible, Planck's method consists in a division of the total 'phase space' of all co-ordinates and momenta into cells, with the help of pairs of surfaces nh apart ($n=1, 2, \ldots$) which are invariant integrals of the equations of motion. In this way he obtained for instance the energy in terms of quantum numbers for the rotator, the symmetric top, the ordinary and relativistic Kepler motion, etc. He even tackled the asymmetric top (1918). Then he applied the results to the optical problem of the rotational spectra of molecules (1917). There he had to overcome a particular difficulty connected with his method of quantization; this allowed for each set of quantum numbers

small but finite domains in the phase space, while the observations showed rather sharp lines. The way in which he removed this apparent contradiction has to-day only historical interest like all the work done in this period. Therefore, it suffices to mention some other papers which show that he always tried to attack the most interesting problems of the day. He calculated the heat of dissociation of the hydrogen molecule according to the 'ring model' suggested by Bohr and Debye (1919). He tried to solve Gibb's paradox of statistical mechanics by a careful determination of the free energy of gas molecules with arbitrary velocity distribution (1922). Several papers under different titles deal with the fluctuations of energy in the black body radiation (1923, 1924). He discussed a difficulty concerning the free energy of atomic hydrogen gas; as Bohr had already noticed the partition function taken over the discontinuous states diverges in this case as the energy values approach zero like $-n^{-1}$. Planck's solution consists essentially in cutting off the discrete spectrum where the radius of the orbit reaches the linear dimensions of the vessel; he does not however neglect the remainder but shows that in these states the electron can be treated as a free particle.

One paper entitled 'A new statistical definition of entropy' (1925) contains a general formulation of Boltzmann's and Gibb's statistical expression of the entropy $S=k \log P$ for quantum systems; Planck defines P as the number of stationary states for which the energy does not exceed a given value E (instead of taking the sum over all states in a given narrow energy interval) and he shows that this leads to the correct expression for a system of oscillators and for a monatomic gas.

When the war ended I left Berlin. Max von Laue, Planck's celebrated pupil, wished to return to Berlin and to be near his master; so he offered me an exchange of my Berlin position (Extraordinariat) with his full professorship in Frankfurt-on-Main, and as Planck agreed, I accepted. From 1919 on Berlin enjoyed this constellation of three most brilliant theoretical physicists, Planck, Einstein, v. Laue, which was soon to be enhanced by a fourth, Schrödinger. He had published in 1926 his paper on wave mechanics which made an immediate impression everywhere, even more than Planck's discovery in 1900; for the world of physics was prepared for this step by the work done during the preceding twenty-five years and in particular by the publications of de Broglie and the Göttingen school.

So it was only natural, that in 1928 when Planck reached his seventieth year and had to resign his Chair, Schrödinger became his successor. Planck, however, did not retire into inactivity. He remained permanent secretary of the mathematical physical class of the Berlin Academy and continued his scientific work and publications, free from the burden of lecturing to students.

Planck had never had a research school, like Sommerfeld in Munich, and the number of pupils who wrote a thesis under his direction is small. I have already mentioned K. von Mosengeil, E. Zermelo and M. von Laue; then there are Max Abraham, known through his book on Maxwell's theory of electricity, F. Reiche who wrote one of the first books on quantum theory,

E. Lamla, H. Kallmann and a few others. Lise Meitner was Planck's assistant for a considerable time.

But large numbers of students have attended his lectures and studied his books. His normal course of lectures was published in 1930 in five volumes, corresponding to five semesters ($2\frac{1}{2}$ years) lecturing. The first four contain mechanics of points and rigid bodies, mechanics of continuous substances, electricity and magnetism, optics; the last volume gives a condensed account of thermodynamics, the theory of radiation and quantum theory. They are the prototype of similar lectures given at all German universities. An English translation has spread their influence over a wider area. Planck has edited books and lectures by Clausius and Kirchhoff. In 1910 he published a series of eight lectures given by him the previous year at Columbia University, New York; in 1922 a book entitled *Physikalische Rundblicke*, and more recently, 1943, a collection of his speeches and addresses in two volumes under the title *Wege zur physikalischen Erkenntnis*.

The last period of his scientific life is that of quantum mechanics. What he expected from the work of his successor is revealed in the address, with which he, as Secretary of the Academy, replied to Schrödinger's inaugural lecture (1929). Planck welcomed wave mechanics as the solution of a crisis threatening physics, namely the sceptical attitude towards the universal validity of the law of causality.

I quote the last words of this address: 'You were the first to show how the spatio-temporal processes in an atomic system can in fact be completely determined, though only under the supposition that one regards as their elements not the motions of particles but of material waves; and how the mysterious discontinuous proper values of the energy of the system can be calculated with absolute accuracy from your differential equation together with natural boundary conditions, while the question about the physical significance of the waves can be left undecided'.

This crisis of causality occupied his mind very much, as is seen from his numerous popular writings and addresses. Before speaking of these it must be mentioned that up to his very old age he continued to publish papers on special subjects, mainly those on which he had worked in earlier periods. There is a series (1930, 1931, 1933) on the boundary layers of dilute electrolytes, one on the principle of le Chatelier and Braun (1934), one on the production of electricity in electrolytes. Most remarkable are three papers with the title 'Attempt at a synthesis between undulatory and corpuscular mechanics' written in 1940, when he was above eighty years of age. They contain a careful consideration of the transition from wave mechanics to particle mechanics through the limiting process $h \to o$. He shows that to obtain this transition an additional condition must be fulfilled, and he postulates this condition to hold rigorously, instead of the usual boundary conditions of the Schrödinger equation. Translated into the language of optics it means the exclusion of all solutions which correspond to diffraction phenomena, as Wessel has pointed out. I do not share Planck's hope that his 'modified wave mechanics' will

bridge the gap between quantum and classical physics; but it shows clearly how deeply Planck was worried by the logical hardships which his own work has imposed on the physicists. This brings me to a short account of his philosophical writings which became more and more numerous with increasing age and predominate in his last period.

It is hardly possible to attach to Planck's work a label with one of the traditional philosophical systems; it has strains of rationalism, idealism, empiricism. But there is one school which he emphatically and repeatedly rejected: positivism. His spirited controversy with Ernst Mach is still worth reading. Planck started it in 1909 with an article on 'the Unit of the Physical Picture of the World', published in the *Physikalische Zeitschrift*. The next volume (1910) of this periodical contains Mach's strongly ironical reply and a final article of Planck which is not less pointed and peppered. Mach defends his idea that all science is due to the principle of economy of thinking which itself can only be understood in the frame of Darwin's biological theory, and he claims to have thus found a basis for science free from all metaphysics. Planck's main answer is that this principle of economy itself is certainly metaphysical. There are many other points of disagreement. Mach was sceptical about the existence of atoms, he declared Boltzmann's kinetic theory, even the absolute zero of temperature, to be unproved hypothesis, and he attacked the Newtonian concept of absolute rotation, anticipating in some vague way Einstein's theory of general relativity. But this is the only point where he was right, in all other questions at issue Planck's physical intuition was confirmed by the later development of physics.

In 1930 Planck renewed his attack against the anti-metaphysical school in a lecture, 'Positivism and the Real External World', in which he presents his arguments in a less caustic but most convincing way. I quote a paragraph containing the essence of this paper:

'The basis given to physics by positivism, though well founded, is too narrow, it has to be widened by an additional statement, whose importance is this: it frees science as far as possible from the incidences produced by the relation to human individuals. And this is achieved through a fundamental step into metaphysics, not imposed by formal logics but by common sense; namely through the hypothesis, that our personal experiences do not form the physical world, but that they only bring us messages from another world which lies behind them and which is independent of them; in other words, that there exists a real external world.'

The same idea appears in many of his philosophical lectures and articles. Their general tendency is to show that science is nothing but developed and refined common sense.

Meanwhile Planck's own child, quantum theory, had grown beyond all expectation and now dominated the whole of physics; but it had taken a direction which led straight away from Planck's fundamental convictions. Causality and strict determinism, even the assumption of an external objective world independent of ourselves became problematic. Planck discussed these

questions in numerous publications, always maintaining the essence of his principles and trying to reconcile them with the facts of physics. Some of these articles culminate in a consideration of the paradoxes connected with the conception of free will in a deterministic world. Planck's solution is this: Determinism holds without exception, and we can use it for predicting not only events in inorganic nature, but even the behaviour of other human beings—though never our own behaviour. For by thinking about our possible decisions we influence them and can therefore not predict them. Hence there is no contradiction between the belief in free will and rigorous causality. An English version of Planck's ideas can be found in the Guthrie Lecture of the Physical Society, London, which he gave in 1932 under the title 'The Concept of Causality', published in the *Proceedings of the Physical Society* and discussed in *Nature* (1932, p. 45). Planck was a religious man and several of his articles deal with the relation of science and faith (1930, 1947). He believed that science could contribute not only to material progress but also to the moral and spiritual development of mankind. There was no gap in his mind between his scientific and religious convictions.

Planck enjoyed good health up to his old age. This was certainly due to the simplicity and regularity of his life and to his custom of having real holidays. He spent the vacations mostly in the Alps, staying some weeks in lonely mountain villages near the high peaks, and then at his little property near Tegernsee. He loved the mountains and was a trained and hardened mountaineer. I visited him once in Trafoi when he was well over sixty; he had just returned from climbing the Ortler, a summit of 12,000 feet.

I was soon to meet him again in South Tyrol under different circumstances. After having been dismissed by Hitler in April 1933, my family and I left Germany at once for a little house in the Dolomites which we had rented for the summer. Planck spent the summer in a neighbouring valley, where I visited him. He told me then that in his capacity as President of the Kaiser Wilhelm Gesellschaft he had to pay a visit to Hitler and tried on this occasion to intervene in favour of his colleague Fritz Haber without whose method of fixing nitrogen from the air the First World War would have been lost by Germany from the beginning. Hitler's reaction was a violent outburst against the Jews in general. He finally brought himself into such a rage that Planck could do nothing but listen silently and take his leave. He later, in 1947, described the scene in *Physikalische Blätter*. After the failure of this attempt to plead for reason and restraint Planck seems to have given up all hope of changing the course of events, and he kept an outward peace with the powers in being. Yet there is no doubt about his true feelings, and the Nazis knew it. Goebbels wrote in his *Diary* (English edition by L. P. Lochner, p. 295): 'It was a great mistake that we failed to win science over to support the new state. That men such as Planck are reserved, to put it mildly, in their attitude towards us, is the fault of Rust (the Minister of Education) and is irremediable.' Planck continued to serve at the Academy, the Kaiser Wilhelm Gesellschaft and other public institutions, with the hope of saving German science and

learning from total destruction. The Prussian tradition of service to the state and allegiance to the Government was deeply rooted in him. I think he trusted that violence and oppression would subside in time and everything return to normal. He did not see that an irreversible process was going on.

Planck has been in this country on several occasions and has had many friends here. In 1937 he came to Scotland to receive an Honorary degree at Glasgow and the honorary membership of the Royal Society of Edinburgh. He and his wife stayed in my house; it was the last time that we discussed matters scientific, political and personal. When I met him after the war at the Newton celebrations of the Royal Society in 1946 he was only a shadow of his former self, tired and frail, yet with his kindly smile unchanged. His house in Grunewald was destroyed in one of the big air raids on Berlin, and he lost everything, including his library. His son Erwin, the only surviving one of the four children of his first marriage, who held a high post in the Government, was involved in the July plot of 1944 against Hitler and was killed by the Nazis.

I know little about Planck's life during the war. He and his wife had found a refuge on the estate of a friend in Rogätz, on the river Elbe, near Magdeburg. There they came between the lines of the retiring Germans and of the advancing Allied armies, the battle raged around them for days. When Pohl, the physicist in Göttingen, heard of their plight he induced the Americans to send a military car and take them to the safety of Göttingen.

Planck bore his Job-like fate with quiet fortitude, resigning himself to the will of God. In this his deterministic philosophy may have helped him as well as his faith. He made his last home in Göttingen, but undertook long and tedious journeys when he was invited to lecture. On one of these occasions he fell seriously ill at Bonn but miraculously recovered from double pneumonia in spite of his eighty-eight years. So it could be hoped that he would reach his ninetieth birthday for which a great celebration was being prepared. Yet a few months before this date he began to fail and died on 4 October 1947 in Göttingen. The planned birthday celebration was changed into a memorial service which took place on 23 April 1948. It was attended by representatives of numerous scientific institutions in Germany and in many other countries.

The list of those institutions which have honoured Planck by awarding him a degree or by electing him a member is too long to be reproduced here. A few only may be mentioned. He had the German degrees of Dr. rer.nat. h.c., Dr. ing. h.c., Dr. med. h.c., also honorary degrees of several British universities, including Cambridge. He was a member of all the German and Austrian Academies (Berlin, Munich, Dresden, Göttingen, Vienna) and of many others (Britain, Denmark, Eire, Finland, Greece, Holland, Hungary, Italy, Russia, Sweden, Ukraine, United States). The Royal Society of London elected him a Foreign Member in 1926. He received the Nobel prize in 1919. One of the small planets was 'given' to him as a present on his eightieth birthday by the astronomers and called Planckiana. In 1930 he became President and in 1946 honorary President of the Kaiser Wilhelm Gesellschaft which has now been renamed 'Max Planck Gesellschaft'.

A Planck Medal has been founded by the German Physical Society, which he was the first to receive. He was awarded the Goethe-Preis of the city of Frankfurt-on-Main in 1946 and was appointed honorary member and 'knight' of an American Mark Twain Society.

Thus his greatness has been acknowledged by his contemporaries. Will posterity confirm this judgment? We who have witnessed the incredible transformation of science which his work has brought about in less than half a century, have no doubt it will.

M. Born

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