Albert Einstein, 1879-1955

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ALBERT EINSTEIN was born on 14 March 1879, at Ulm in Wurttemberg, Germany, the son of Hermann Einstein and his wife, Pauline, née Koch. Hermann Einstein was in partnership with his brother, an engineer, in a small factory producing electrical supplies. From an autobiographical fragment* we learn that the Einstein parents were Jewish, but non-observant of their religion; at a very early age the boy's religious nature became dissatisfied with the spiritual emptiness of his surroundings: seeking for something deeper, he attached himself ardently for a time to the faith of his fathers; but further reading led him to the opinion that fact cannot easily be separated from legend in the framework of Jewish history, and he ceased to accept Judaism as a transcendental religion, while retaining its humanitarian principles. In later life he was a keen Zionist and a Governor of the Hebrew University of Jerusalem.

Meanwhile, his education was somewhat irregular, owing chiefly to changes in domicile brought about by unsatisfactory circumstances in his father's business. Between the ages of 10 and 15 he attended the Luitpold Gymnasium at Munich. Later for a year he became a pupil of the cantonal school at Aarau in Switzerland, and at the age of 17 he entered the Technische Hochschule at Zurich, each successive move involving some discontinuity in the curriculum. Thrown very much on himself, he read a great many popular books of science and general knowledge, and was deeply impressed by the principle of the reign of law in the universe. This now became the centre of his religion. 'I believe', he declared, 'in Spinoza's God, who reveals himself in the harmony of all being, not in a God who concerns himself with the fate and actions of men.' The conception of a God who controls the grand mechanism of the world, but is indifferent to the petty affairs of human beings, recalls the system of Aristotle, in which the Prime Mover causes the rotation of the heavenly spheres, but takes no interest in their inhabitants. This view induced in Einstein what he calls a 'cosmic religious feeling', that is, a belief in the possibility of interpreting nature in terms of a mathematical system of great beauty and simplicity. This is the spiritual background of his mature intellectual achievements. By 1901 Einstein had completed his studies at Zurich Polytechnic School and had taken the step of becoming a Swiss citizen. He then obtained a position in the Patent Office in Berne, and married a lady, Mileva Maric, who had been a student with him at Zurich. She was a native of Hungary, of Serbian race, and nominally

of the Greek Orthodox religion. Two sons, Hans Albert and Eduard, were born to them on 14 May 1904 and 26 July 1910 respectively: they were baptized as Greek Catholics, but educated without reference to any particular religion.

In the years 1901 to 1904 Einstein published some investigations on molecular forces and thermodynamic principles, in particular, two papers (3) and (4)*, in which he obtained independently certain results on the statistical kinetic theory of heat which had been published a year or two earlier by Willard Gibbs. In 1905, (7), continued in (11), he applied these results to the motion of very small particles suspended in a liquid. The particles were supposed to be much larger than a molecule, but it was assumed that as a result of collisions with the molecules of the water, they acquire a random motion, like that of the molecules of a gas. The average velocity of such a suspended particle even in the case of particles large enough to be seen with a microscope, might be of observable magnitude, but the direction of its motion would change so rapidly, under the bombardment to which it would be exposed, that it would not be directly measurable. However, as a statistical effect of these transient motions, there would be a resultant motion which might be within the range of visibility. Einstein showed that in a finite interval of time $t$ the mean square of the displacement for a spherical particle of radius $a$ is

$$\frac{RTt}{3\pi a \mu N}$$

where $R$ is the gas constant, $T$ the temperature, $N$ is Avogadro’s number, and $\mu$ is the coefficient of viscosity. Thus, by this phenomenon the thermal random motion, hitherto a matter of hypothesis, might actually be made a matter of visible demonstration.

The motion of small particles suspended in liquids had been observed as early as 1828 by Robert Brown, a botanist, after whom it was called the Brownian motion. Einstein identified the motion studied by him with the Brownian motion, somewhat tentatively at first, but without hesitation in (11). The direct confirmation of the kinetic theory provided by studies of the Brownian movement was the means of converting to it some notable former opponents, such as Wilhelm Ostwald and Ernst Mach.

In the same volume of the Annalen der Physik as (7), Einstein, now aged 26, published two other papers which attracted unusual attention. In the first of these, (6), he supposed monochromatic radiation of absolute temperature $T$, frequency $\nu$ or $c/\lambda$ and small energy density $\rho$ (within the range of values of $\nu/T$ for which Wien’s formula of radiation

$$\rho = \frac{8\pi c h}{\lambda^5} \exp\left(-\frac{hc}{k\lambda T}\right) \, d\lambda$$

is applicable where $h$ is Planck’s constant and $k$ is Boltzmann’s constant) to be

* The numbers refer to the bibliography at the end of this notice.
contained in a hollow chamber of volume $v_0$ with perfectly reflecting walls, its total energy being $E$: and, investigating by use of Wien’s formula the dependence of the entropy on the volume, he found for the difference of the entropies when the radiation occupies the volume $v_0$ and when it occupies a smaller volume $v$, the equation

$$S - S_0 = \frac{Ek}{hv} \ln \frac{v}{v_0}.$$  

Now, by inverting the Boltzmann–Planck relation

$$\text{entropy} = k \times \text{logarithm of probability},$$

he calculated the relative probability from the difference of entropies, and found that the probability that at an arbitrarily-chosen instant of time, the whole of the energy of the radiation should be contained within a part $v$ of the volume $v_0$, is

$$(v/v_0)^{E/hv}.$$  

This formula he studied in the light of a known result in the kinetic theory of gases, namely that if a gas contained in a volume $v_0$ consists of $n$ molecules, the probability that at arbitrarily-chosen instant of time, all the $n$ molecules should be collected together within a part $v$ of the volume, is

$$(v/v_0)^n.$$  

Comparing these formulae, he inferred that the radiation behaves as if it consisted of $E/hv$ quanta of energy or photons* each of amount $hv$. The probability that all the photons are found at an arbitrary instant in the part $v$ of the volume $v_0$ is the product of the probabilities $(v/v_0)$ that a single one of them is in the part $v$; which shows that they are completely independent of each other.

Now, the quantum theory had originated in the year 1900, when Max Planck had asserted that a vibrator of frequency $\nu$ can emit or absorb energy only in multiples of $hv$. Planck at that time regarded the quantum property as belonging essentially to the interaction between radiation and matter: free radiation he supposed to consist of electromagnetic waves, in accordance with Maxwell’s theory of 1861-4. Einstein in this paper put forward the hypothesis that parcels of radiant energy of frequency $\nu$ and amount $hv$ occur not only in emission and absorption, but that they have an independent existence in the aether.

It was shown by several writers within a few years that Einstein’s hypothesis leads not to Planck’s law of radiation but to Wien’s, at any rate if it is assumed that each of the light quanta or photons of frequency $\nu$ has energy $hv$ and that they are completely independent of each other. In order to obtain Planck’s law, it is necessary to assume that the elementary photons of energy $hv$ form aggregates, or photo-molecules as we may call them, of energies $2hv$, $3hv$, . . ., respectively, and that the total energy of radiation is distributed on the

* The word photon was actually introduced much later, namely by G. N. Lewis, *Nature, Lond.*, 18 December, 1926; but it is so convenient that we shall adopt it now.
average, in a regular manner between the photons and the different kinds of photo-molecules.

Einstein applied his idea in order to construct a theory of photo-electricity. In 1899 J. J. Thomson and P. Lenard had shown independently that a metal irradiated by ultra-violet light emits negative electrons; and in 1902 Lenard, continuing his researches, had shown that the number of electrons liberated is proportional to the intensity of the incident light, so long as its frequency remains the same, and that the initial velocity of the electrons is altogether independent of the intensity of the light, but depends on its frequency.

In Einstein's paper of 1905 (6) he asserted that when a metal surface is illuminated by radiation, the radiation consists of parcels of energy; when one such parcel or photon falls on the metal, it may be absorbed and liberate a photo-electron; and that the maximum kinetic energy of the photo-electron at emission is \( (hv - e\phi) \), where \( v \) is the frequency of the light, and \( e\phi \) is the energy lost by the electron in escaping from its original location to outside the surface. This, of course, implies that no photo-electrons will be generated unless the frequency of the light exceeds a certain 'threshold' value \( e\phi/h \).

Einstein's equation was verified experimentally in 1912-16. For many metals, the threshold frequency is in the ultra-violet; but for the electro-positive metals, such as the alkali metals, it is in the visible spectrum: for sodium, it is in the green. The discoveries of this paper, and its sequel (12), were named as the chief grounds for the award of the Nobel Prize to Einstein in 1922.

We now pass to (8), the third of the famous papers of 1905. Natural Philosophy, in the eighteenth and nineteenth centuries, was founded on Newton's laws of motion. According to the First Law, any particle which is free from the action of impressed forces, moves, if it moves at all, with uniform velocity in a straight line. But in order that this statement may have a meaning, it is necessary to define the terms straight line and uniform velocity: for a particle which is said to be 'moving in a straight line' in a terrestrial laboratory would not appear to be moving in a straight line to an observer on the sun, since he would perceive its motion compounded with the earth's diurnal rotation and her annual revolution in her orbit. We can, however, define a straight line with reference to a system of axes \( Oxxyz \) as the geometrical figure defined by a pair of linear equations between \( x, y, z \), and we can assert as a fact of experience that certain systems of axes \( Oxxyz \) exist such that free particles move in straight lines with reference to them. Moreover, we can assert that there exist certain ways of measuring time such that the velocity of free particles along their rectilinear paths is uniform. A set of axes in space and a system of time-measurement, which possess these properties may be called an inertial system of reference.

In Newtonian mechanics, if \( S \) is an inertial system of reference, and if \( S' \) is another system such that the axes \( O'x'y'z' \) of \( S' \) have any uniform notion of pure translation with respect to the axes \( Oxxyz \) of \( S \), and if the system of time-measurement is the same in the two cases, then \( S' \) is also an inertial system.
of reference; the Newtonian laws of motion are valid with respect to $S'$ just as with respect to $S$. No one inertial system of reference can be regarded as having a privileged status, in the sense that it could properly be said to be fixed, while the others were moving; so long as mechanics only is considered, Newtonian physics does not involve the notion of the absolute fixity of a point in space.

But physics comprised also optics; and the undulatory theory of optics, which was universally accepted in the second half of the nineteenth century, involves the doctrine of a medium, the aether, which permeates all space. Apart from its small vibrations, which constitute light, the aether must be supposed to be at every place in some state of motion or rest; and thus the question arose, what motion has the aether relative to (say) the moving earth at any point?

In the later years of the nineteenth and earlier years of the twentieth century experimental attempts were made to answer this question, based on physics as it was then understood, but all were unsuccessful: and in his lectures at the Sorbonne in 1899, Henri Poincaré said 'I regard it as very probable that optical phenomena depend only on the relative motions of the material bodies, luminous sources, and optical apparatus concerned'; in other words, he believed that absolute motion is indetectable in principle, by either dynamical or optical means. In the following year, at an International Congress of Physics held in Paris, he elevated this into a new principle introduced into physics, which would resemble the Second Law of Thermodynamics, inasmuch as it asserted the impossibility of doing something: in this case, the impossibility of determining the velocity of the earth relative to the aether. In a lecture to a Congress of Arts and Science at St Louis, U.S.A., on 24 September 1904, Poincaré gave to a generalized form of this principle the name The Principle of Relativity. ‘According to the Principle of Relativity,’ he said, ‘the laws and physical phenomena must be the same for a “fixed” observer as for an observer who has a uniform motion of translation relative to him; so we have not, and cannot possibly have, any means of discerning whether we are or are not carried along in such a motion.

We must now enquire how an analytic scheme can be devised which will enable physics to be reformulated in accordance with Poincaré’s Principle of Relativity. In particular, we must learn how observers who have uniform motions of translation relative to each other can express the laws of the electromagnetic field in the same form, a discovery that was made in 1903 by Lorentz.*

Denoting by $\mathbf{d}$ the electric vector, by $\mathbf{h}$ the magnetic vector, and by $c$ the velocity of light, the fundamental equations of the aether in empty space may be written

\[
\begin{align*}
\text{div } \mathbf{d} &= 0, \\
\text{curl } \mathbf{d} &= -\partial \mathbf{h}/\partial t \\
\text{div } \mathbf{h} &= 0, \\
\text{curl } \mathbf{h} &= \partial \mathbf{d}/\partial t
\end{align*}
\]

Now consider the transformation of the co-ordinates $t, x, y, z$ which is represented by the equations

\[
\begin{align*}
ct &= ct_1 \cosh \alpha + x_1 \sinh \alpha, \\
x &= x_1 \cosh \alpha + ct_1 \sinh \alpha, \\
y &= y_1, \\
z &= z_1,
\end{align*}
\]

where $\alpha$ denotes a constant parameter. The origin of the $(t_1, x_1, y_1, z_1)$ system has at the instant $t$ the co-ordinates $x = ct \tanh \alpha, y = 0, z = 0$, so it is moving along the axis of $x$ with the constant velocity $c \tanh \alpha$. Moreover, transform $\mathbf{d}, \mathbf{h}$, to new variables $\mathbf{d}_1, \mathbf{h}_1$, according to the equations

\[
\begin{align*}
d_x &= d_{x,1}, & h_x &= h_{x,1}, \\
d_y &= d_{y,1} \cosh \alpha + h_{y,1} \sinh \alpha, & h_y &= h_{y,1} \cosh \alpha + d_{x,1} \sinh \alpha, \\
d_z &= d_{z,1} \cosh \alpha - h_{x,1} \sinh \alpha, & h_z &= h_{z,1} \cosh \alpha + d_{y,1} \sinh \alpha
\end{align*}
\]

Then in terms of these new variables the differential equations of the aether become

\[
\begin{align*}
\text{div}\, \mathbf{d}_1 &= 0, & c \text{ curl}\, \mathbf{d}_1 &= -\partial \mathbf{h}_1 / \partial t_1, \\
\text{div}\, \mathbf{h}_1 &= 0, & c \text{ curl}\, \mathbf{h}_1 &= \partial \mathbf{d}_1 / \partial t_1
\end{align*}
\]

that is to say, the equations in terms of the new variables have the same form as the original equations. In this transformation (1) the variable $x$ plays a privileged part, as compared with $y$ or $z$. We can, of course, at once write down similar transformations in which $y$ or $z$ plays the privileged part; and we can combine any number of these transformations by performing them in succession. The aggregate of all the transformations so obtained, combined with the aggregate of all the rotations in ordinary space, constitutes a group, to which Poincaré* gave the name the group of Lorentz Transformations.

Einstein in (8) adopted Poincaré's Principle of Relativity (using Poincaré's name for it) as a new basis for physics and showed that the group of Lorentz transformations provided a new analysis connecting the physics of bodies in motion relative to each other. Notable results appearing in this paper for the first time were the relativist formulae for aberration and also for the Doppler effect.

In a later memoir, (9), of the same year Einstein discussed another consequence of relativity theory. In 1881 J. J. Thomson had shown that an electrically charged spherical conductor moving in a straight line behaves as if it had an additional mass of amount $(4/3c^2)$ times the energy of its electrostatic field.† In 1900 Poincaré, referring to the fact that in free aether the electromagnetic momentum is $(1/c^2)$ times the Poynting flux of energy, suggested that electromagnetic energy might possess mass density equal to $(1/c^2)$ times the energy density: that is to say, $E = mc^2$ where $E$ is energy and $m$ is mass.

† It was shown long afterwards by Fermi that the transport of the stress system set up in the material of the sphere should be taken into account, and that when this is done, Thomson's result becomes additional mass $= (1/c^3)$ energy of field.
In (9) and also in some later papers (13), (19) and (20) Einstein supported this conclusion, which both in atomic physics and in astrophysics has proved to be of immense importance.

The papers of 1905 made such an impression that Einstein was widely recognized as a man worthy of exalted academic position, and after becoming a Privatdozent at the University of Berne he was in 1909 appointed 'Professor Extraordinary' of Theoretical Physics in the University of Zurich, and in the autumn of the following year, Professor of Theoretical Physics in the German University of Prague. This was one of the two parts, German and Czech, into which the ancient University of Prague had, in 1888, been divided, for political reasons.

Einstein was now continually trying to remove certain imperfections which he considered to be present in the Theory of Relativity as it had existed in 1906; and in 1907 and 1911 he published papers, (20) and (41), in which he introduced what he later called the Principle of Equivalence, which may be thus described: Consider an observer who is enclosed in a chamber without windows, so that he is unable to find out by direct observation whether the chamber is in motion relative to an outside world or not. Suppose the observer finds that any object in the chamber, whatever be its chemical or physical nature, when left unsupported, falls towards one particular side of the chamber with an acceleration $f$ which is constant. The observer would be justified in putting forward either of two alternative explanations to account for this phenomenon. (i) He might suppose that the chamber is 'at rest', and that there is a field of force, like the earth's gravitational field, acting on all bodies in the chamber, and causing them if free to fall with acceleration $f$; or (ii) he might explain the observed effects by supposing that the chamber is in motion, if he postulates that in the outside world there are co-ordinate axes $(C)$ relative to which there is no field of force, and if he moreover supposes the chamber to be in motion relative to these axes $(C)$ with an acceleration equal in magnitude but opposite in direction to $f$; then it is obvious that free bodies inside the chamber would have an acceleration $f$ relative to the chamber.

The observer has no criterion enabling him to tell which of these two explanations is the true one. If we could say definitely that the chamber is at rest, then explanation (i) would be true, while if we could say definitely that the axes $(C)$ are at rest, then explanation (ii) would be true. But by the Principle of Relativity, we cannot give a preference to one of these sets of axes over the other: we cannot say that one of them is moving and the other at rest; and we must, therefore, regard the two explanations as equally valid or, in other words, must assert that a homogeneous field of force is equivalent to an apparent field which is due to the accelerated motion of one set of axes relative to another—a uniform gravitational field is physically equivalent to a field which is due to a change in the co-ordinate system.

In (20) Einstein also showed, by combining Doppler's principle with the principle of equivalence, that a spectral line generated by an atom situated
at a place of very high gravitational potential, e.g. at the sun’s surface, has, when observed at a place of lower potential, e.g. on the earth, a greater wavelength than the corresponding line generated by an identical atom on the earth. This may be shown very simply as follows. If we denote by $\Omega$ the gravitational potential at the sun’s surface, the energy lost by a photon of frequency $v$ in escaping from the sun’s gravitational field is $\Omega h v / c^2$. Remembering that the energy $hv$ is $hc/\lambda$, we see that the wavelength of the solar radiation as measured by the terrestrial observer is $1 + (\Omega/c^2)$ times the wavelength of the same radiation when produced on earth.

In 1911 Einstein followed up this work by (41), in which he argued that since light is a form of (electromagnetic) energy, therefore, light must gravitate,* that is, a ray of light passing near a powerful gravitating body such as the sun, must be curved; and the velocity of light must depend on the gravitational field.

At this time also he made an important contribution (43) to his photon theory of light, by extending it to photochemical theory. Developing a suggestion made by Stark in 1908, he now asserted that when chemical action takes place under the influence of light, a molecule being dissociated as a result of absorbing radiation of frequency $v$, the amount of energy absorbed by the molecule is $hv$.

His life was also affected by another academic change: he was offered and accepted a Chair of Theoretical Physics in his Alma Mater, the Polytechnic School at Zurich and he removed to Zurich in the summer of 1912.

In 1912-3 a number of theories of gravitation were published by Abraham, Einstein, (44), (45), (47), (48), Nordstrom and Mie; the decisive step forward was taken in 1913 by Einstein when he adopted a method of characterizing the state of space near a point, which was familiar to differential geometers and had been used in physics in 1909 by Harry Bateman.† According to Bateman, the condition that a luminous disturbance originating at the space point $(x^1, x^2, x^3)$ at the instant $t^0$, should arrive at the space point $(x^1 + dx^1, x^2 + dx^2, x^3 + dx^3)$ at the instant $(t^0 + dt^0)$ could be expressed as the vanishing of a quadratic differential form which may be written

$$\sum_{p,q=0}^{3} g_{pq} dx^p dx^q$$

when the coefficients $g_{pq}$ are functions of $(t^0, x^1, x^2, x^3)$. This form (1) is invariant for all transformation of the co-ordinates $(t^0, x^1, x^2, x^3)$: and its coefficients $g_{pq}$ are characteristic of the physical properties of space near the point.

In empty space free from all forces the quadratic differential form (1) reduces to

$$c^2(dl)^2 - (dx)^2 - (dy)^2 - (dz)^2,$$

where $c$ is the velocity of light, since the vanishing of this expression is the

condition that a luminous disturbance originating at the point \((x, y, z)\) at the instant \(t\) should arrive at the point \((x+dx, y+dy, z+dz)\) at the instant \((t+dt)\).

It had been shown in the later years of the nineteenth century by Ricci and Levi-Civita that when the physical conditions are such that the vanishing of the quadratic differential form \((2)\) must be replaced by the vanishing of the more general differential form \((1)\) then the differential calculus for operations in this space is replaced by a more general calculus called the absolute differential calculus or tensor-calculus. During his stay in Prague Einstein developed a close friendship with a mathematician, named Georg Pick, who expressed the opinion that the mathematical instrument needed for the further development of relativity theory was precisely tensor calculus; and Einstein now saw how this idea could be carried out. For, writing

\[
(ds)^2 = c^2(d\tau)^2 - (dx)^2 - (dy)^2 - (dz)^2,
\]

we know that the rectilinear motion of a particle free from all forces in empty space is determined by the equation

\[
\delta \int ds = 0,
\]

where \(\delta\) is the symbol of the calculus of variations. Einstein now, with the help of his old friend Marcel Grossmann in Zurich, (51), (54), (55), (63), (65), put forward the theory, that similarly the motion of a free material particle in any gravitational field is determined by the equation

\[
\delta \int ds = 0,
\]

where

\[
(ds)^2 = \sum_{p,q=0}^{3} g_{pq} \, dx^p \, dx^q
\]

is Bateman's quadratic form.

This was a tremendous innovation, because it implied the abandonment of the time-honoured belief that a gravitational field can be specified by a single scalar potential function: instead, it proposed to specify the gravitational field by the ten functions \(g_{pq}\) which could now be spoken of as the gravitational potentials. Einstein justified* this new departure by showing that the theory of a single scalar gravitational potential led to unacceptable inferences. He compared, for instance, two systems, in the first of which a movable hollow box with perfectly-reflecting walls is filled with pure-temperature radiation, while in the second the same radiation is contained inside a fixed vertical pit which is closed at the top and bottom by movable pistons connected by a rod so as to be always at a fixed distance apart, the pit walls and pistons all being perfectly-reflecting; and he showed that on the

* In §7 of (51).
single-scalar-potential theory the work necessary to raise the radiation upwards against the force of gravity would in the second system be only one-third of the work required in the first system, a conclusion which was obviously wrong. He admitted, however, that in his own mind the strongest reason for rejecting the single-scalar-potential theory was his conviction that relativity in physics exists not only with respect to the Lorentz group of linear orthogonal transformations, but with respect to a much wider group.

The ten coefficients $g_{pq}$ not only specify the force of gravitation, but they determine also the scale of distance in every direction and the rate of clocks. The metric defined by

$$(ds)^2 = \sum_{p,q=0}^{3} g_{pq} dx^p dx^q$$

is not, in general, Euclidean; and since its non-Euclidean qualities determine the gravitational field, we may say that gravitational theory is reduced to geometry, in accordance with an idea expressed by FitzGerald* in 1894 in the words ‘Gravity is probably due to a change of structure of the aether, produced by the presence of matter.’ The ‘aether’ of FitzGerald was called by Einstein simply ‘space’ or ‘space-time’; and FitzGerald’s somewhat vague term ‘structure’ became with Einstein the more precise ‘curvature’. Thus we obtain the central proposition of the Einsteinian theory: ‘Gravity is due to a change in the curvature of space-time, produced by the presence of matter.’

In comparing FitzGerald’s statement with Einstein’s, it may be remarked that if we consider a gravitational field which is statical, i.e. such as would be produced by gravitating masses that are permanently at rest relative to each other, then feeble† electromagnetic phenomena taking place in it can be shown to happen exactly in accordance with the ordinary Maxwellian theory of electromagnetic phenomena taking place in a medium whose specific inductive capacity and magnetic permeability are anisotropic and vary from point to point.

It is possible that when FitzGerald said ‘Gravity is probably due to a change of structure of the aether’, he was actually thinking of a change which would show itself in alterations of the dielectric constant and magnetic permeability, and that he had in mind an electrical constitution of matter, on account of which matter would be subject to forces depending on the values of the dielectric constant and magnetic permeability: by analogy with the fact that in a liquid whose dielectric constant varies from point to point, an electrified body moves from places of lower to places of higher dielectric constant.

What differentiates the Einsteinian theory from all previous conceptions is that the older physicists had regarded gravity as merely one among many types of natural force—electric, magnetic, etc.—each of which influenced in its own way the motion of material particles. Space, whose properties were

* FitzGerald’s Works, p. 313.
† i.e. so feeble that they do not appreciably change the curvature of the field.
set forth in Euclidean geometry, was, so to speak, the stage on which the forces played their parts. But in the new theory gravity was no longer one of the players, but part of the structure of the stage. A gravitational field consisted essentially in a replacement of the Euclidean properties by a much more complicated kind of geometry: space was no longer homogeneous or isotropic. An analogy may be drawn from the game of bowls. Bowling-greens, in the north of England, are not flat, but rise to a slight elevation in the centre. An observer who failed to notice the central elevation would find that a bowl (supposed without bias) always described a path convex toward the centre of the green, and he might account for this by postulating a centre of repellent force there. A better-informed observer would attribute the phenomenon to a geometrical feature—the slope. The two explanations correspond respectively to the Newtonian and the Einsteinian conceptions of gravity: for Newton it is a force, for Einstein it is a modification of the geometry of space.

When the metric of space-time is specified by an equation

$$(ds)^2 = \sum_{p,q=0}^{3} g_{pq} dx^p dx^q,$$

an observer moving in any manner will have a world-line consisting of the points of space-time which he successively occupies; and at any point of his world-line he will have in his immediate neighbourhood an instantaneous three-dimensional space, formed by the aggregate of all the elements of length which are orthogonal to his world-line at the point, orthogonality being defined by the statement that two vectors $(X)$ and $(Y)$ are said to be orthogonal if

$$\sum_{p=0}^{3} X_p Y^p = 0,$$

where $(X_p)$ is the covariant form of one vector and $(Y^p)$ is the contravariant form of the other.

Einstein laid down the principle that the equations which describe any physical process must satisfy the condition that their covariance with respect to arbitrary substitutions can be deduced from the invariance of $ds$. In other words, the laws of nature must be represented by equations which are covariant for the form

$$\sum_{p,q=0}^{3} g_{pq} dx^p dx^q$$

with respect to all point-transformations of co-ordinates. Laws of nature are assertions of coincidences in space-time, and, therefore, must be expressible by covariant equations.
It might be thought that by following up the consequences of this principle we should obtain important positive results. However, Ricci and Levi-Civita had shown long before that from practically any assumed law we can derive another law which does not differ from it in any way that can be tested by observation, but which is covariant. The fact that a formula has the covariant property does not, therefore, tell us anything as to whether it is correct or not. We are, however, perhaps justified in believing that a conjectural law which can be expressed readily and simply in covariant form is more worthy of attention (as being more likely to be true) than one whose covariant form is awkward and complicated.

Not only must the general laws of physics be covariant, it is also necessary that every single assertion which has a physical meaning must be covariant with respect to arbitrary transformations of the co-ordinate system. Thus the assertion that an electron is at rest for an interval of time of duration unity cannot have a physical meaning, since this assertion is not covariant.

In Einstein's general theory, the velocity of light at any place has always the value \( c \) with respect to any inertial frame of reference for this neighbourhood, and the velocity of any material body is less than \( c \). Thus there is no difficulty in the fact that the fixed stars have velocities greater than \( c \) with respect to axes fixed in the rotating earth: for such axes are not inertial.

Some physicists called attention to the fact that when light is propagated in a medium where there is anomalous dispersion, the index of refraction may be less than unity, whence it seemed as if the velocity of light in the dispersive medium might be greater than the velocity of light in vacuo. The difficulty was removed when it was pointed out by L. Brillouin and A. Sommerfeld that the velocity of light with which the index of refraction is concerned is the phase velocity, whereas the velocity of a signal is the group velocity, which is never greater than \( c \).

It has sometimes been supposed, by a misunderstanding, that the general Einsteinian theory requires us to regard the Copernican conception of the universe as no more true than the Ptolemaic, and that it is indifferent whether we regard the Earth as rotating on her axis or regard the stellar universe as performing a complete revolution about the earth every twenty-four hours. The root of the matter, by which everything is explained, is that the Copernican axes are inertial, while the Ptolemaic are not. The earth rotates with respect to the local inertial axes.

In his paper (51), published in 1913, Einstein (at p. 241) gave the form which Maxwell's equations of the electromagnetic field must take in a gravitational field, i.e. when the matric of space-time is given by a quadratic differential form

\[
(dx)^2 = \sum_{\mu, \nu} g_{\mu \nu} dx^\mu dx^\nu.
\]

If \( d \) is the electric vector and \( h \) is the magnetic vector, then we know that
Maxwell’s equations in Euclidean spaces consist of the Ampère–Maxwell tetrad
\[
\begin{align*}
\frac{\partial d_x}{\partial x} + \frac{\partial d_y}{\partial y} + \frac{\partial d_z}{\partial z} &= 4\pi \rho, \\
\frac{\partial h_x}{\partial y} - \frac{\partial h_y}{\partial z} &= \frac{1}{c} \frac{\partial d_z}{\partial t}, \\
\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x} &= \frac{1}{c} \frac{\partial d_y}{\partial t}, \\
\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} &= \frac{1}{c} \frac{\partial d_z}{\partial t}, \tag{A}
\end{align*}
\]
and the Faraday tetrad
\[
\begin{align*}
\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} &= 0, \\
\frac{\partial d_x}{\partial y} - \frac{\partial d_y}{\partial z} &= -\frac{1}{c} \frac{\partial h_x}{\partial t}, \\
\frac{\partial d_x}{\partial z} - \frac{\partial d_z}{\partial x} &= -\frac{1}{c} \frac{\partial h_y}{\partial t}, \\
\frac{\partial d_y}{\partial x} - \frac{\partial d_x}{\partial y} &= -\frac{1}{c} \frac{\partial h_z}{\partial t}, \tag{B}
\end{align*}
\]
Now write \(x^0 = ct, x^1 = x, x^2 = y, x^3 = t:\)
then the electric and magnetic vectors together constitute a six-vector
\[
d_x = X^{01}, d_y = X^{02}, d_z = X^{03}, h_x = X^{23}, h_y = X^{31}, h_z = X^{12},
\]
and the Ampère–Maxwell tetrad of equations written in covariant form are
\[
\Delta iv \left( X^{pq} \right) = 4\pi \mathcal{F}^p, \tag{A}
\]
where \(\Delta iv \left( X^{pq} \right)\) denotes the vectorial divergence of \(X^{pq}\), that is
\[
\sum_{q=0}^{3} (X^{pq})_{q},
\]
where the suffix outside the bracket denotes covariant differentiation, and where \(\mathcal{F}^p\) denotes the four-vector which represents the electric charge and current; and the Faraday tetrad written in covariant form is
\[
\Delta iv \left( Y^{pq} \right) = 0, \tag{B}
\]
where \(Y^{pq}\) is the six-vector dual to \(X_{pq}\), i.e. such that
\[
Y_{pq} = \sqrt{(-g)} \ X^{rs},
\]
where \((p, q, r, s)\) is an even permutation of the numbers \((0, 1, 2, 3)\). The
The six-vector of the electromagnetic field may be expressed in terms of potentials, in the same way as in Euclidean space it is expressed by the equations

\[
\begin{align*}
\frac{\partial \phi}{\partial x} - \frac{1}{c} \frac{\partial a_x}{\partial t} = h_x &= \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}.
\end{align*}
\]

etc. For if we write \((\phi_0, \phi_1, \phi_2, \phi_3)\) for \((\phi, -a_x, -a_y, -a_z)\), these equations become

\[
X_{pq} = (\phi_p)_q - (\phi_q)_p,
\]

where the suffixes outside the brackets represent covariant differentiation: and we see that the potential \((\phi_0, \phi_1, \phi_2, \phi_3)\) is a covariant vector.

Electromagnetic theory lends naturally to physical optics, and this again to geometrical optics. Now in the relativity theory of Poincaré and Lorentz, for which the line-element \(ds\) in the world of space-time is given by

\[
(ds)^2 = (d\tau)^2 - \left( \frac{dx^2 + dy^2 + dz^2}{c^2} \right)
\]

the geodesics of the world are straight lines, and the null geodesics (i.e. the geodesics for which \(ds\) vanishes) are the straight lines for which

\[
\frac{(dx)^2 + (dy)^2 + (dz)^2}{(d\tau)^2} = c^2;
\]

so the null geodesics are the tracks of rays of light. When Einstein created his new general theory of relativity, in which gravitation was taken into account, he carried over this principle, and asserted its truth for gravitational fields. The principle was, however, not proved at that time: and indeed there was the obvious difficulty in proving it, that strictly speaking there are no 'rays' of light—that is to say, electromagnetic disturbances which are filiform, or drawn out like a thread—except in the limit when the frequency of the light is infinitely great; in all other cases diffraction causes the 'ray' to spread out.

The matter was investigated in 1920 by M. von Laue, who, starting from the partial differential equations for electromagnetic phenomena in a gravitational field, obtained a particular solution which corresponded to light of infinitely high frequency, and showed that the path of this disturbance satisfied the differential equations of the null geodesics: thus for the first time proving the truth of Einstein's assertion. It was afterwards shown by E. T. Whittaker that the law is really an immediate deduction from the theory of the characteristics of partial differential equations, and that it is not necessary to introduce the notion of frequency at all: in fact, that in a gravitational field, any electromagnetic disturbance which is filiform must necessarily have the form of a null geodesic of space-time.

In September 1913 Einstein lectured on 'gravitation' to the Physics Section of the Naturforschenversammlung at its fifth meeting, held in Vienna.
the discussion that followed, it was clear that many German men of science were not yet converted to his ideas. Doubts were expressed regarding the validity of his views on the equality of inertial and gravitational mass, on the velocity of propagation of gravitational processes, on the possibility of ever being able to detect the deflexion of light rays in a gravitational field, and on the predicted red-shift of spectral lines in such a field. Dr R. W. Lawson, who was present, recalls that the older generation was more sceptical than the younger men, some of whom later became distinguished workers in special and general relativity.

In 1913 it was decided to invite Einstein to settle in Berlin as a member of the Royal Prussian Academy of Sciences in connexion with the Kaiser Wilhelm Gesellschaft, which had recently been founded by Wilhelm II as a centre of research institutes. Max Planck and Walther Nernst, who were at the time the leading German physicists, journeyed to Zurich to convince Einstein of the desirability of the plan, and persuaded him to accept it; he left Zurich for Berlin at the end of 1913.

Not long after making the change he separated from his wife Mileva Maric. He now became attached to another lady, his cousin, Elsa, whom he had known as a child in Munich. She was the daughter of a business man, and was now a widow with two daughters. The marriage to Mileva was dissolved on 14 February 1919, although in 1924 she formally obtained leave to continue to use the name Einstein. She and Einstein's two sons continued to live in Switzerland. Mileva never left Switzerland, and died in Zurich on 4 August 1948; the elder son became a civil engineer, and eventually settled in America. He is now professor of hydraulic engineering at the University of California, Berkeley. The younger son had a nervous breakdown at about the age of 20, from which he never recovered. Elsa was married to Einstein and lived with him as his wife until her death in 1936.

In the papers that have been referred to, which are of date earlier than November 1915, Einstein gave a satisfactory account of the behaviour of mechanical and electrical systems in a field of gravitation which is supposed given: his formulae were derived fundamentally from the principle of equivalence, i.e. the principle that the systems behave just as if there was no gravitational field, but they were referred to a co-ordinate system with an acceleration equal and opposite to the acceleration of gravity. But he had not as yet succeeded in obtaining an entirely satisfactory set of fundamental equations for the gravitational field itself, i.e. equations which would play the same part in his theory that Poisson's equation,

\[
\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi \rho,
\]

played in the Newtonian theory. This defect was repaired, and the theory (now known as General Relativity) substantially completed, in a series of short papers, (68), (69), (70), published in November–December 1915, in the Berlin Sitzungsberichte.
Let us first inquire what expressions covariant with respect to the form
\[ \sum_{p,q} g_{pq} dx^p dx^q \]
can be formed from the \( g_{pq} \) and their derivatives alone. It can be shown that these can all be derived from a certain tensor of rank 4, known as the Riemann tensor, which is denoted by \( K_{pqrs} \). From the Riemann tensor we can obtain a tensor of rank 2 which is defined by the equation
\[ K_{pq} = \sum_{r,s} g^{rs} K_{pqrs}; \]
it is called the Ricci tensor or contracted-curvature tensor and from the Ricci tensor we obtain what is called the scalar curvature of the space, defined by the equation
\[ K = \sum_{p,q} g^{p\bar{q}} K_{\bar{p}q}. \]

For a two-dimensional space, e.g. a surface in Euclidean three-dimensional space, \(-\frac{1}{2}K\) is the ordinary Gaussian measure of curvature.

Now Mach had long before introduced a principle, that inertia must be reducible to the interaction of bodies; Einstein generalized this into what he called Mach's principle, namely that the field represented by the ten potentials \( g_{pq} \) is determined solely by the masses of bodies. The word ‘mass’ is here to be understood in the sense given to it by the theory of relativity, that is, as equivalent to energy. Now energy is expressed covariantly by an energy-tensor of rank 2, introduced in 1908 by Hermann Minkowski, which is denoted by \( T_{pq} \): so in the fundamental equations of gravitation, corresponding to the equation \( \nabla^2 V = -4\pi\beta\rho \) of the Newtonian theory (where \( \beta \) is the Newtonian constant of gravitation), we may expect the tensor \( T_{pq} \) or some linear function of it, to take the place of Poisson’s \( \rho \). We expect to find on the other side of the equation, corresponding to \( \nabla^2 V \), a tensor of the same rank as \( T_{pq} \) that is, the second rank, containing second derivations of the potentials, but no higher derivatives. The only covariant tensors of this character are the Ricci tensor \( K_{pq} \) with \( K_{p\bar{q}}, \) \( g_{p\bar{q}} \) and \( g_{pq} \). Einstein first supposed that \( K_{pq} \) might be a simple constant multiple of \( T_{pq} \): but this is not satisfactory, since the divergence of \( T_{pq} \) is zero and the divergence of \( K_{pq} \) is not in general zero: and he finally proposed the equations
\[ K_{pq} = -\kappa (T_{pq} - \frac{1}{2}g_{pq} T) \]
where \( T = \sum_{p,q} g^{p\bar{q}} T_{p\bar{q}} \)
and \( \kappa \) is a constant depending on the Newtonian constant of gravitation. These are the general field-equations of gravitation.
Multiplying them by $g^{pq}$, summing with respect to $p$ and $q$, and remembering that
\[ \sum_{p,q} g^{pq} g_{pq} = 4 \]
we have $K = \kappa T$, so the equations may be written
\[ K_{pq} - \frac{1}{2} g_{pq} K = -\kappa T_{pq} \quad (p,q = 0, 1, 2, 3). \]
These are ten equations for the ten unknowns $g_{pq}$: there are four identities between them, as might be expected, for four of the $g_{pq}$ can be assigned arbitrarily as functions of the $x^2$, corresponding to the fact that the equations are invariant under the most general transformation of co-ordinates.

In the complete form of the theory as developed later, material bodies are regarded as singularities in the field, i.e. regions where the gravitational field-equations $K_{pq} = 0$ are not satisfied; and eventually he proved that the 'equations of motion' which determine the motion of these singularities in presence of each other can be deduced from the field equations.

According to Mach's principle as adopted by Einstein, the curvature of space is governed by physical phenomena, and we have to ask whether the metric of space-time may not be determined wholly by the masses and energy present in the universe, so that space-time cannot exist at all except in so far as it is due to the existence of matter. The point at issue may be illustrated by the following concrete problem: if all matter were annihilated except one particle which is to be used as a test-body, would this particle have inertia or not? The view of Mach and Einstein was that it would not; and in support of this view it may be urged that, according to the deductions of general relativity, the inertia of a body is increased when it is in the neighbourhood of other large masses: it seems needless, therefore, to postulate other sources of inertia, and simplest to suppose that all inertia is due to the presence of other masses. When we confront this hypothesis with the facts of observation, however, it seems that the masses of whose existence we know—the solar system, stars, and nebulae—are insufficient to confer on terrestrial bodies the inertia that they actually possess; and, therefore, if Mach's principle were adopted, it would be necessary to postulate the existence of enormous quantities of matter in the universe which have not been detected by astronomical observation, and which are called into being simply in order to account for inertia in other bodies. This is, after all, no better than regarding some part of inertia as intrinsic.

The relation of the constants in Einstein's and Newton's laws of motion is readily found: the constant $\kappa$ of the Einsteinian theory is connected with the Newtonian constant, the $\beta$ of Poisson's equation $\nabla^2 \psi = -4\pi\beta\rho$, by the equation
\[ \kappa = \frac{8\pi\beta}{c^2}. \]
In (69), the third of his papers in the *Berlin Sitzungsberichte* of 1915,
Einstein showed that the new gravitational theory could explain an anomaly that had long been known to affect the motion of the perihelion of the planet Mercury, namely that the line of apsides advances 43" in a century. It was shown much later by H. R. Morgan\* that the earth’s perihelion also has a secular motion, much smaller in amount, which agrees with the amount calculated by Einstein’s theory. Other explanations of these perihelion effects are, however, not altogether impossible.

A second comparison of General Relativity with observation proposed by Einstein, was that a ray of light coming from a star and passing close to the sun’s gravitational field, when observed by a terrestrial observer, should be deflected through an angle of about 1.75". This prediction was tested at the solar eclipse of May 1919, and was at the time regarded as confirmed observationally; but later eclipses gave somewhat different results, and the matter must be regarded as still unsettled.

A third observational test proposed was the displacement to the red of spectral lines emitted in a strong gravitational field: here, however, complications are introduced by other possible factors. On the whole question of the comparison of General Relativity with observation, as it was regarded in the early days cf. E. Wiechert, *Ann. Phys.*, *Lpz.* 63, 301-381 (1920).

The philosophy of science exhibited in Einstein’s work had some novel and characteristic features. He rejected the view of the British empiricists, that knowledge in physics is obtained straightforwardly by applying the methods of abstraction and induction to facts ascertained by experience: and on the other hand he rejected the teaching of Kant, that there are certain self-evident truths, such as the axioms of geometry, which are derived from reason alone, independently of experience. The Einsteinian doctrine is that a mathematician, contemplating Nature, can find systems of mathematical equations which satisfy certain conditions and are worthy of trial as furnishing possibly correct physical theories. One such condition is invariance: the meaning of the equations is not to be affected by any arbitrary change in the choice of co-ordinates. Another condition is simplicity: he had (like Kepler) a mystical belief that the simplest equations are the most likely to be true; and yet another condition is that in special cases they should reduce to results already known to be true in those cases. Systems of equations of immense generality can in this way be found and afterwards confronted with the facts of observation.

Philosophically, the work of Einstein has obviously much in common with Descartes’s doctrine of vortices; in both cases the theory was entirely a mental construction proposed for examination as a possible explanation of experience; and doubtless much of the opposition that Einstein encountered was due to a survival of the disfavour into which cartesianism had by this time fallen. Some of it may have been due to the popular principle attributed to Rutherford, that an alleged scientific discovery has no merit unless it can be explained to a barmaid. In 1916 Einstein published (75) and (82), a new and extremely

simple proof of Planck's law of radiation, and at the same time obtained some important results regarding the emission and absorption of light by molecules. The train of thought followed was more or less similar to that adopted by Wien in the derivation of his law of radiation, but Einstein now adapted it to the new situation created by Bohr's theory of spectra.

Consider a molecule of a definite kind, disregarding its orientation and translational motion: according to quantum theory, it can take only a discrete set of states $Z_1, Z_2, \ldots, Z_n, \ldots$, whose internal energies may be denoted by $e_1, e_2, \ldots, e_n, \ldots$. If molecules of this kind belong to a gas at temperature $T$, then the relative frequency $W_n$ of the state $Z_n$ is given by the formula of Gibbs's canonical distribution as modified for discrete states, namely (for simplicity omitting consideration of the statistical 'weight' of the state)

$$W_n = e^{-e_n/kT}.$$ 

Now suppose that the probability of a single molecule in the state $Z_m$ passing in time $dt$ spontaneously, i.e. without excitation by external agencies (as in the emission of $\gamma$-rays by radioactive bodies) to the state of lower energy $Z_n$ with emission of radiant energy $e_m - e_n$ is

$$A_n dt \quad (A)$$

Suppose also that the probability of a molecule under the influence of radiation of frequency $\nu$ and energy-density $\rho$ passing in time $dt$ from the state of lower energy $Z_n$ to the state of higher energy $Z_m$ by absorbing the radiant energy $e_m - e_n$ is

$$B_m \rho dt \quad (B)$$

and suppose that the probability of a molecule under the influence of this radiation-field passing in time $dt$ from the state of higher energy $Z_m$ to the state of lower energy, $Z_n$, with emission of the radiant energy $e_m - e_n$, is

$$B_n \rho dt \quad (B')$$

This is called stimulated emission; its existence was recognized here for the first time.

Now the exchanges of energy between radiation and molecules must not disturb the canonical distribution of states as given above. So in unit time, on the average, as many elementary processes of type (B) must take place as of types (A) and (B') together. We must therefore have

$$e^{-e_n/kT} B_m \rho = e^{-e_m/kT} (B_m \rho + A_m)$$

We assume that $\rho$ increases to infinity with $T$, so this equation gives

$$B_m^n = B_m^n \quad (1)$$

and the preceding equation may, therefore, be written

$$\rho = \frac{(A_m^n/B_m^n)}{\exp{[(e_m - e_n)/kT]} - 1}.$$
This is evidently Planck's law of radiation: in order that it may pass asymptotically into Rayleigh's law for long wavelengths, and into Wien's law for short wavelengths, we must have

\[ \epsilon_m - \epsilon_n = h\nu \]

and

\[ A_m^r = \frac{8\pi h\nu^3}{c^3} B_m^r. \]  

(2)

The two equations (1) and (2), first given in this paper of Einstein's, are fundamental in the theory of the exchanges of energy between matter and radiation, and have been extensively used in the later development of quantum theory.* The formulae were expanded to the case of non-sharp energy-levels by R. Becker,† and to the laws of interaction between radiation and free electrons by Einstein & Ehrenfest (122).

Another important result established in this paper related to exchanges of momentum between molecules and radiation. Einstein showed that when a molecule, in making a transition from the state \( \mathcal{Z}_n \) to \( \mathcal{Z}_m \), receives the energy \( \epsilon_m - \epsilon_n \), it receives also momentum of amount \( (\epsilon_m - \epsilon_n)/c \) in a definite direction; and, moreover that when a molecule, in the transition from \( \mathcal{Z}_m \) to the state of lower energy \( \mathcal{Z}_n \), emits radiant energy of amount \( (\epsilon_m - \epsilon_n) \), it acquires momentum of amount \( (\epsilon_m - \epsilon_n)/c \) in the opposite direction. Thus the processes of emission and absorption are directed processes; there seems to be no emission or absorption of spherical waves.

In 1916 and the following years Einstein gave much attention, specially (78), (91), (172), to the propagation of disturbances in a gravitational field. If the distribution of matter in space is changed, e.g. by the circular motion of a plate in its own plane, gravitational waves are generated which are propagated outwards with the speed of light. If such waves impinge on an electron which is at rest, the principle of equivalence shows that the physical situation is the same as if the electron were moving with a certain acceleration, and therefore an electron exposed to gravitational waves must radiate.

In 1917 Einstein pointed out, (83), that the field equations of gravitation, as he had given them in 1915, do not satisfy Mach's principle, according to which no space-time could exist except in so far as it is due to the existence of matter (or energy). Einstein's equations of 1915, however, admit the particular solution

\[ g_{pq} = \text{constant}, \quad T_{pq} = 0, \quad (p, q = 0, 1, 2, 3), \]

so that a field is thinkable without any energy to generate it. He, therefore, proposed now to modify the equations by writing them

\[ K_{pq} - \frac{1}{2} g_{pq} K - \lambda g_{pq} = -\kappa T_{pq} \quad (p, q = 0, 1, 2, 3) \]

The effect of the \( \lambda \) term is to add to the ordinary gravitational attraction

* If the weights of the energy-levels are \( g_n, g_m \), the relation (1) must be replaced by \( g_n B_m^r = g_m B_n^r. \)

Relation (2) is unaffected.

† *Z. Phys.* 27, 173 (1924).
between particles a small repulsion from the origin varying directly as the
distance; at very great distances this repulsion will no longer be small, but
will be sufficient to balance the attraction; and in fact, as Einstein showed, it
is possible to have a statical universe spherical in the spatial co-ordinates
with the uniform distribution of matter in exact equilibrium. This is generally
called the Einstein universe.* The departure from Euclidian metric is measured
by the radius of curvature $R_0$ of the spherical space, and this is connected
with the total mass $M$ of the particles constituting the universe by the equation
\[
\gamma M \frac{1}{c^2} = \frac{1}{2} \pi R_0,
\]
where $\gamma$ denotes the Newtonian constant of gravitation. The total volume
of this universe is $2\pi^2 R_0^3$. The cosmological problem when the $\lambda$-term is not
introduced was discussed in 1931 in (157).

After 1918 there were many exhibitions of feeling in Germany hostile to
Einstein, led by the eminent physicist Philipp Lenard of Heidelberg, who
was an early member of Hitler’s party. The Prussian Minister of Education,
Hänisch, wrote to Einstein begging him to take no notice of these attacks,
with such effect that Einstein, who in Imperial Germany had retained his
Swiss nationality, now became a German citizen.

The development of relativity after 1918 was greatly influenced by a
theory put forward in that year by Weyl,† in which the attempt was made
to represent electromagnetic forces, as gravity was represented, as con­
sequences of the pure geometry of space and time. Weyl’s geometry, instead
of being specified like Einstein’s by a single quadratic differential form
\[
\Sigma g_{pq} dx^p dx^q
\]
was specified by a quadratic differential form
\[
\Sigma g_{pq} dx^p dx^q
\]
and a linear differential form
\[
\Sigma \phi_p dx^p
\]
together. The coefficients $g_{pq}$ of the quadratic form can be interpreted as in
Einstein’s theory as the potentials of gravitation, while the four coefficients
$\phi_p$ of the linear form can be interpreted as the four components of the electro­
magnetic potential-vector. Thus Weyl succeeded in exhibiting both gravita­
tion and electricity as effects of the metric of the world. Einstein wrote on this
theory in (106) (1921), demurring to the physical validity of some of Weyl’s
considerations.

* The suggestion that our universe might be an Einsteinian space-time of constant spatial curvature
seems to have been first made by Ehrenfest in a conversation with De Sitter about the end of 1916.
The enlargement of geometrical ideas thus achieved by Weyl was followed in 1921 by a still wider extension due to Eddington, which was based on an analysis of the notion of parallelism in curved space, due to T. Levi-Civita. In 1923 Eddington’s theory was further developed (118), (119), (120), by Einstein. The primitive quantities in this theory, in terms of which the spacetime manifold and its properties are described, are not the fourteenmetrical and electrical tensor-components $g_{pq}$, $\phi$, but forty quantities $\Gamma_{\mu\nu}^a$ which specify the parallel-transport and are called components of the affine field. A contracted Riemann tensor of the second rank is built up from these quantities in the usual way, and is then split into symmetric and skew parts which are identified respectively with the gravitational $g_{pq}$ and the electrical $\phi$. In these papers Einstein obtained the forty equations which determine the $\Gamma_{\mu\nu}^a$ as functions of the $g_{pq}$ and the $\phi$, from an Action principle $\delta \int H dt = 0$, where $H$ is a scalar density and the variation is with respect to the $r^v$ as independent variables.

Weyl’s proposal for a unified theory of gravitation and electromagnetism was followed up in another direction by Th. Kaluza, in whose system the ten gravitational potentials $g_{pq}$ of Einstein and the four components $\phi$ of the electromagnetic potential were expressed in terms of the line-element in a space of five dimensions, in such a way that the equations of motion of electrified particles in an electromagnetic field became the equations of geodesics. This theory was discussed approvingly by Einstein in 1927 (138). In 1921 Einstein was invited by Chaim Weizmann, the leader of the Zionist movement, who was a lecturer in organic chemistry in the University of Manchester, to accompany him on a journey to the United States, where they were received with tremendous enthusiasm. In returning, he visited England.

In 1924 Satyandra Nath Bose of Dacca University gave a new derivation of Planck’s formula of radiation, based on the assumption that radiation is composed of photons, which for statistical purposes can be treated like the particles of a gas, but with the important difference that photons are indistinguishable from each other, so that instead of considering the allocation of individual distinguishable photons among a set of states, he fixed attention on the number of states that contain a given number of photons. Bose’s paper showed that photons obey a particular kind of statistics, different from the classical statistics of Maxwell, and that the recognition of this fact leads immediately to Planck’s law of radiation. Einstein seems to have translated this paper into German from an English manuscript sent to him by Bose, and immediately extended it, (125) and (131), to the study of a monatomic ideal gas; a photon (apart from its polarization property) differs from a monatomic molecule only in that its static mass is vanishingly small. The average number

† R.C. Circ. mat. Palermo, 42, 173.
of particles of mass $m$ in unit volume of such a gas, with energies in the range $\epsilon$ to $\epsilon + d\epsilon$, is

$$\frac{2\pi}{\hbar^3} \frac{(2m)^3}{e^{e/(kT)} - \mu - 1} \epsilon^4 d\epsilon$$

where $\mu$ is a constant. This is the fundamental formula of what is generally called Bose–Einstein statistics.

In 1928 Einstein introduced, (140), (141), (145), (146), (151), (152), (153), (154), (158), a proposal for a unitary theory of the gravitational and electromagnetic fields, by devising a Riemannian geometry with new invariants and tensors which retained the notion of parallelism between line-elements at a distance from each other, as in Euclidean geometry.* In (159) (1931) the concept of parallelism at a distance was dispensed with. In (160) (1932) he discussed the line-element of space-time applicable to the universe as a whole, when it is assumed that at every instant the spatial section is flat and the cosmical constant $\lambda$ is zero, showing how, from observations of the velocity of recession of the nebulae, the mean density of matter in the universe might be estimated.

For the winter of 1932 the Einsteins went to the California Institute of Technology at Pasadena. In January 1933 Adolf Hitler was appointed Chancellor of the German Reich and the 'purge' of Jews began. Einstein went back to Europe, but not to Germany; in the spring of 1933 he settled in a Belgian town near Ostend and wrote to Planck resigning his position in Berlin. In October 1933 he sailed from Southampton for New York, to accept a position in the Institute for Advanced Study, being appointed a member of the Institute for life. Later he acquired American citizenship. At Princeton he published joint papers with many disciples.

A unified field theory somewhat different from that of 1928, in which the fundamental tensor was taken to be Hermitian, was developed in 1945-8, (185), (187), (189); and yet another, announced in the press at Christmas 1949, was published in 1950 (192): this made use of a non-Riemannian geometry based on a fundamental tensor $g_{\mu\nu}$, which is analyzed into a symmetric and a skew part: and was further modified in (194), (195).

The appeal of General Relativity has in recent years been weakened by a growing doubt as to whether continuous differential equations in four-dimensional space-time can possibly provide a solution of some of the problems of quantum theory, such as those relating to the time of decay of a radium atom. This has been the subject of many papers. Einstein took one side, holding that the world can be completely described by a field theory of the type of General Relativity, which is rigidly deterministic; cf. (167), (169). Most other theoretical physicists, particularly Max Born, Niels Bohr, and Wolfgang Pauli, maintain on the contrary that in micro-physics strict causality must be abandoned. The matter is discussed by many writers in

* Accounts of this theory are given by Eddington in *Nature, Lond.* 123, 280-1 (23 February 1929), and by Einstein in *Ann. Inst. Poincaré*, 1, 1-24 (1931).
the volume *Albert Einstein: Philosopher-Scientist*, edited by P. A. Schilpp. The last paper which had Einstein as its sole author (196), was devoted to this question of the interpretation of quantum-mechanics.

Einstein died on 18 April 1955. For much help in the preparation of this notice I am indebted to his son, Professor Hans Albert Einstein, and to Professor Dr E. Stiefel of Zürich. The photograph is by Karsh of Ottawa.

**EDMUND WHITTAKER**

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